Performance Evaluation for Multi-arm Manipulation of Hollow Suspended Organs
Wei Wei, Student Member, IEEE, Roger E. Goldman, Howard F. Fine, Stanley Chang, and Nabil Simaan, Member, IEEE

Abstract—This paper presents a unified mathematical framework for modeling and evaluating the performance of multiple robotic arms that operate on hollow suspended organs. This framework is applied to a novel two-armed hybrid robotic system being developed for ophthalmic vitreous surgeries. Four cases are designated to capture the general movements required for any surgical procedure associated with hollow suspended organs. Dexterity measures, based on multiple characteristic lengths, are presented for procedures corresponding to these manipulation cases. Simulation results of the dual-arm robotic system for ophthalmic surgery are presented for all four manipulation cases. A comparison of this robotic system with current surgical tools shows a significant improvement in intraocular dexterity.

Index Terms—Dexterity, hybrid robots, medical robotics, multiple-arm manipulation, ophthalmic surgery.

I. INTRODUCTION

Due to the micron scale and delicacy of retinal tissues, only highly experienced ophthalmic surgeons can successfully perform these demanding, yet common, procedures. Robot-assisted ophthalmic surgery has been proposed to address this challenge. Grace [1] presented a miniature telesurgical parallel robot for the treatment of retinal venous occlusion in ophthalmic surgery. Das et al. [2] and Charles et al. [3] developed robot assisted micro surgery (RAMS) workstation, which is a cable-driven master/slave telerobotic system. Ang et al. [4] developed a hand-held microsurgical instrument for vitreoretinal microsurgery via tremor canceling. Taylor et al. and Kumar et al. [5], [6] presented a cooperative manipulation robot for microsurgical applications. Ikuta and Kato [7] designed a hand-held forceps with an active joint and fiberscope to address the lack of dexterity inside the eye.

Previous works addressed major challenges of statistically characterizing hand tremor [8], [9], providing active tremor cancellation [4], high precision [1], [2], [6], and force monitoring in eye surgery [10]. However, the surgical need to manipulate several instruments with high precision in the vitreous body, while stabilizing or manipulating the eye under the microscope presents a novel integrated challenge. This need led to our recent paper [11] and ongoing development of a robotic system that is capable of intraocular and orbital manipulations of the eye (see Fig. 1).

The problem of stabilizing the eye while manipulating structures within [11] can be generalized to any operation on hollow organs with a stability not guaranteed by anatomic constraints, e.g., eye, stomach, bladder, etc., as illustrated in Fig. 2. The suspended organ is manipulated and stabilized using several robotic arms that are inserted into the organ. The literature addresses the modeling and evaluation of constrained multibody system [12]–[15] and multifingered hands [16]–[18]. However, these mathematical models cannot be directly applied to systems, as in Fig. 2, due to assumption in the grasping literature that the stabilizing bodies do not penetrate the grasped object.

Performance evaluation of this system required an updated metric due to its hybrid structure. Although many works have focused on performance measures including manipulability...
ellipsoids [13], [19], [20], manipulability polytopes [21], [22], isotropy measures [23], [24], kinematic conditioning [25]–[27], and dexterity [28], [29], these metrics do not address the problem of dexterity evaluation for systems with multiple parallel and serial sections. Wang and Chirikjian presented the error propagation problem of hybrid robots in [30]; however, an effective method to evaluate the hybrid robot's performance is still missing. Furthermore, there is no work done to systematically model and evaluate the dexterity of surgical procedures on hollow suspended organs while considering the dexterity in manipulating the organ and operating inside it.

The contributions of this paper are the following. A unified kinematic model is developed to analyze a partially constrained hollow object manipulated by several inserted robotic arms. This model is applicable to robotic assistance on hollow suspended organs, such as the eye. The paper applies this framework to justify the design of a dual-arm telemanipulator slave for ophthalmic surgery. This slave is aimed at answering the unaddressed challenge of orbital manipulation and dexterous bi-manual operations inside the eye. Further, our approach adds to existing work on dexterity evaluation of multifingered hands and cooperative manipulation systems by analyzing the constrained motion of a hollow object through four manipulation cases. These cases model the dexterity in manipulating the object and operating inside it via insertion constraints. The paper defines dexterity measures for each of these four cases and identifies the clinical relevance of each case in ophthalmic surgery.

II. CURRENT SURGICAL SETUP FOR OPHTHALMIC SURGERY

The typical surgical setup for ophthalmic surgery is shown in Fig. 3. Two surgeons coordinate to perform the ophthalmic surgical procedures. The main surgeon sits superior to the patient’s head and performs most of the surgical tasks, including manipulation of the surgical instruments and the light source. The assistant surgeon sits beside the patient’s head to provide irrigation and removal of fluids and adjust the placement of the external visualization lenses. A second assistant delivers tools and supplies to the surgeons.

Three incisions are typically made in the sclera to provide access to the vitreous body for an irrigation tube, a surgical instrument, and a light source. The light source occasionally performs double duty as a second surgical tool. The irrigation tube injects liquid to maintain intraocular pressure. The light source illuminates the retina for proper visualization under the microscope. The surgical tools, including picks, forceps, vitrectomy cutters, and other laser ablation devices, vary depending on the requirements of the procedure [31]. The surgeons operate using a microscope while visualizing the retina through a dilated iris. Because the visual field does not contain the entire retinal surface, procedures often require tilting the eye under the microscope in order to view peripheral areas of the retina.

The repertoire of manipulation tasks in ophthalmic surgical procedures is segmented into four cases of intraocular and orbital manipulations (see Fig. 4). Although Fig. 4 describes these cases with reference to ophthalmic surgery, they are applicable to manipulation of any hollow suspended organ.

Case 1: Intraocular operation with eye stabilization. This case quantifies the ability of the robotic system to perform a specified surgical task inside the eye (Fig. 4(a)).

Case 2: Eye manipulation with constrained intraocular motions. This case evaluates orbital dexterity, a measure of how well the robotic arms can rotate the eye, while respecting kinematic constraints at the incision points and maintaining zero velocity of the forceps relative to the retina (Fig. 4(b)).

Case 3: Eye manipulation with unconstrained intraocular motion. This case evaluates the orbital dexterity without constraint of the forceps relative to the retina (Fig. 4(c)).

Case 4: Simultaneous eye manipulation and intraocular operation. This case measures dexterities of simultaneous intraocular operation and orbital manipulation as the robot rotates the eye and operates inside it (Fig. 4(d)).

The relevance of these cases in ophthalmic surgery is depicted in Figs. 5 and 6. Membrane peeling (Fig. 5(a)) falls into case 1. In this procedure, the eye must be stabilized by the arms while dexterous and precise reach is performed in peeling the micron-thick membrane of scar tissue away from the underlying retina. Fig. 5(a) shows a bi-manual approach for membrane peeling, while Fig. 6(a) shows a forceps used to pull a membrane.

The visualization inside the eye is limited by the dilated iris. The surgeons must tilt the eye under the microscope in order to visualize the peripheral areas of the eye. This movement is performed while steadying the micro-tools inside the eye, and therefore, falls into case 2. Examples shown in Figs. 5(b) and 6(b) illustrate membrane peeling by the ciliary body. This procedure presents challenges in accessing and visualizing the surgical site using straight tools while tilting the eye.
Fig. 5. Illustration of medical procedures corresponding to the four manipulation cases defined in Fig. 4. These figures have been reproduced based on [31] with permission of the American Medical Association (AMA).

Fig. 6. Intraoperative photographs of procedures that perform analogous maneuvers to cases 1–4 in Fig. 4. (a) Central epiretinal membrane peeling over the macula requires stabilization of the eye while gently peeling thin membranes. (b) Peripheral epiretinal membrane peeling in close proximity to the ciliary body requires tilting of the eye for visualization. (c) Silicone oil is removed by tilting the eye so it floats upward toward the aspirating cannula. (d) Complex dissection of membranes in proliferative vitreoretinopathy requires manipulation of both the eye for visualization and the intraocular tools to perform delicate peeling procedures.

Case 3 is relevant for applications where the manipulation of the eye itself is important, while there is no need for intraocular dexterity. An example of such a procedure includes treatment of retinal detachment using a dense liquid (perfluorocarbon liquid) (Fig. 5(c)). In this procedure, a small incision is made in the detached region to allow aspiration of unwanted fluid between the retina and the retinal pigment epithelium. The eye is then tilted in order to roll the liquid on the surface of the retina. The dense liquid is used to flatten the detached region of the retina. Fig. 6(c) shows the application of removing silicone oil by tilting the eyeball.

Case 4 (Figs. 5(d) and 6(d)) is relevant for operations within the peripheral areas of the eye while having to rotate the eye in order to obtain a better view. In current practice, surgeons retract their tools while tilting the eye and subsequently reposition the tip of the instruments in the surgical site. We plan on using the robot to allow simultaneous tilting and distal intraocular operation in order to decrease the operation time and the number of delicate regrasping operations.

The limitations of the current surgical setup include the lack of distal dexterity inside the eye, limited ability to perform precise coordinated bimanual operations, lack of precise stabilization and manipulation of the eye itself under the microscope, limited depth perception, and lack of force feedback. These limitations place stringent constraints on ophthalmic surgeons and prevent them from performing complex bimanual tasks.

III. PROPOSED SURGICAL SETUP AND SYSTEM DESCRIPTION

This paper addresses the need to provide surgeons with a stand-alone surgical system that is capable of manipulating or stabilizing the eye while providing bimanual intraocular dexterous operations. The goal of this system is to enable operations in difficult-to-reach areas of the retina. Examples of surgical procedures that benefit from these capabilities include membrane peeling in anterior aspects of the retina, and dual arm and complex manipulation of blood vessels.

Fig. 1 shows the proposed telerobotic slave. Each arm is composed of a 2 Degrees-of-Freedom (DoF) intraorgan dexterity robot (IODR) and a 6-DoF parallel robot. The IODR is introduced because potential applications require additional DoF inside the organ, which is beyond the ability of available 4-DoF minimally invasive surgical tools.

The IODR utilizes a preshaped NiTi tube that bends in 1 DoF as it is extended outside of a straight cannula with a bending radius of 5 mm (see Fig. 7), similar to the work of [32] and [33] on steerable needles. The length of the IODR cannula is chosen as 60 mm based on the length of existing tools. The IODR is connected to the parallel robot via an adjustable lockable universal joint and a connector link, allowing the IODR cannula having two tilting DoFs so that the angle between the cannula and the connection link, as well as the orientation of the connection link with respect to the moving platform is adjustable. With this design, surgeons can initially position and lock the robot at a given “start” configuration, minimizing the required workspace of the parallel robot.

For the parallel robot, we propose the Stewart–Gough platform design [34] due to its rigidity, compactness, positional accuracy, and high payload-to-weight ratio. These advantages were utilized in other robotic systems for medical applications, e.g., [1] and [35]–[39]. The parallel robot is designed to be placed sideways relative to the patient to avoid singularities.
associated with large tilting angles of the moving platform (Hunt’s singularity [40]).

The dimensions of the parallel robot of each hybrid robotic arm were given in our previous work [11]. These dimensions were chosen to ensure that using both arms allows reaching all points on the retina and to provide orbital manipulation.

IV. MULTI-ARM MANIPULATION OF HOLLOW ORGANS

To fill a hole in the literature addressing constraints that pierce an object, we extend our framework developed in [11] to manipulate and operate on general hollow suspended organs. This general mathematical model applies to all robotic systems with similar constraints.

The instantaneous kinematics of the multiarm robot and the suspended organ is presented in this section. The hollow organ is assumed to be a rigid object movable in 6 DoFs and manipulated by several arms piercing it. Each piercing point presents a 2-DoF constraint specified by maintaining a fixed fulcrum point on the organ. Therefore, for an unconstrained 6-DoF object, three arms (2 DoFs each) are required to fully constrain the system.

For a specific hollow organ, upon introducing the anatomic constraints to the general model, we can obtain the corresponding simplified kinematic model. For example, the orbital musculature of the eye can be modeled constraining the eye to only three rotational DoFs; therefore, it only needs two arms to fully constrain the system. Other hollow suspended organs, like the bladder and stomach, can also be modeled after projecting from the 6-D general space to their individual motion space1.

A. Kinematic Nomenclature

Fig. 8 depicts the organ and the i-th hybrid robotic arm. The organ is enlarged for a clearer view of the end-effector (EE) and the organ coordinate frames. The following coordinate systems are defined to assist in the derivation of the system kinematics. The world coordinate system \( \{ W \} \) is centered at an arbitrarily predetermined point in the patient’s forehead with the patient in a supine position. The \( \hat{z}_W \)-axis points vertically and \( \hat{y}_W \)-axis points superiorly. The Stewart–Gough base coordinate system \( \{ B_i \} \) of the i-th hybrid robot is located at point \( b_i \) (the center of the base platform) such that the \( \hat{z}_{B_i} \)-axis lies perpendicular to the base of the Stewart–Gough platform, and the \( \hat{x}_{B_i} \)-axis lies parallel to \( \hat{z}_W \). The moving platform coordinate system of the i-th hybrid robot \( \{ P_i \} \) lies in the center of the moving platform at point \( p_i \) such that the axes lie parallel to \( \{ B_i \} \) when the Stewart–Gough platform is in the home configuration.

The Stewart–Gough extension arm coordinate system of the i-th hybrid robot \( \{ Q_i \} \) is attached to the distal end of the arm at point \( q_i \), with \( \hat{z}_{Q_i} \), lying along the direction of the insertion needle of the robot \( q_i \hat{n}_i \), and \( \hat{y}_{Q_i} \) fixed during setup procedure. The IODR base coordinate system of the i-th hybrid robot \( \{ N_i \} \) lies at point \( n_i \) with the \( \hat{z}_{N_i} \)-axis also pointing along the insertion needle length \( q_i \hat{n}_i \) and the \( \hat{y}_{N_i} \)-axis rotated from \( \hat{y}_{Q_i} \) by an angle \( q_{s_i,1} \) about \( \hat{z}_{N_i} \). The EE coordinate system \( \{ G_i \} \) lies at point \( g_i \) with the \( \hat{z}_{G_i} \)-axis pointing in the direction of the EE gripper and the \( \hat{y}_{G_i} \)-axis parallel to the \( \hat{y}_{N_i} \)-axis. The organ coordinate system \( \{ O \} \) sits at the rotating center \( o \) of the organ with axes parallel to \( \{ W \} \) when the organ is unactuated by the robot. The additional notations used are defined as the following.

\[ \{ A \} \]
a right-handed coordinate frame with \( \{ \hat{x}_A, \hat{y}_A, \hat{z}_A \} \) as its associated unit vectors and point \( a \) as the location of its origin;

\[ A \overrightarrow{b} \]
vector from point \( a \) to point \( b \) expressed in frame \( \{ A \} \);

\[ [b \times] \]
skew-symmetric cross-product matrix [23] of vector \( b \);

\[ i \]
the index identifying each robotic arm;

\[ \dot{q}_{P_i} = [\dot{q}_{P_{1,i}}, \dot{q}_{P_{2,i}}, \dot{q}_{P_{3,i}}, \dot{q}_{P_{4,i}}, \dot{q}_{P_{5,i}}, \dot{q}_{P_{6,i}}]^T \]
the active joint speeds of the i-th Stewart–Gough platform;

\[ \dot{q}_{s_i} = [\dot{q}_{s_{i,1}}, \dot{q}_{s_{i,2}}]^T \]
joint speeds of the i-th IODR.

The first component is the rotation speed about the axis of the IODR cannula, and the second component is the bending angular rate of the preshaped tube (Fig. 8(b));

\[ A R_B \]
rotation matrix of frame \( \{ B \} \) relative to frame \( \{ A \} \);

\[ \nu_{A/B} \omega_{A/B} \]
relative linear and angular velocities of frame \( \{ A \} \) relative to \( \{ B \} \), expressed in \( \{ C \} \). Unless specifically stated, all vectors in this paper are expressed in \( \{ W \} \);
absolute linear and angular velocities of frame \{A\};

\begin{align*}
\mathbf{v}_A, \omega_A & = \mathbf{W}(\mathbf{a}) \\
\mathbf{\dot{x}}_A, \mathbf{\dot{x}}_P, \mathbf{\dot{x}}_o & = \mathbf{\omega}_A \\
\mathbf{W}(\mathbf{a}) & = \begin{bmatrix} I_{3 \times 3} & [-\mathbf{a}] \times \end{bmatrix} \quad \text{twist transformation operator.}
\end{align*}

**B. System Modeling**

The kinematic modeling of the system has to include the kinematic constraints of the incision points on the hollow organ. The following section presents the kinematics of the triple-armed robot with the organ and describes the kinematics of the IODR following section presents the kinematics of the triple-armed robot with the organ and describes the kinematics of the IODR.

1) **Hybrid Robot:** The Jacobian of the Stewart–Gough platform relating the twist of the moving platform \(x_P\) to the joint speeds \(\dot{q}_i\) is given in (1) [41]. The overall hybrid Jacobian matrix for one robotic arm was obtained as in (2). Appendix A provides derivations of these matrices

\begin{align*}
\mathbf{J}_P & = \mathbf{J}_P(\mathbf{a}) = \left[ \dot{x}_P - \mathbf{T}_x \right] \mathbf{J}_h, \\
\mathbf{\dot{x}}_G & = \mathbf{\dot{q}}_h. \\
\end{align*}

2) **Organ:** Most organs are not free-floating, but rather partially constrained by their surrounding anatomy. A 6-D twist vector is used to describe the motion of the organ

\begin{align*}
\mathbf{x}_o & = [\mathbf{X}_o^T, \mathbf{\omega}_o^T]^T = \begin{bmatrix} \mathbf{x}_{ol} & \mathbf{x}_{oo} \end{bmatrix}^T \\
\mathbf{x}_{ol} & = \begin{bmatrix} x, y, z, \alpha, \beta, \gamma \end{bmatrix}^T \\
\mathbf{x}_{oo} & = \begin{bmatrix} \dot{x}, \dot{y}, \dot{z}, \dot{\alpha}, \dot{\beta}, \dot{\gamma} \end{bmatrix}^T
\end{align*}

where \(x, y, z\) are linear positions and \(\alpha, \beta, \gamma\) are linear velocities of the organ, and \(\mathbf{x}_{ol}\) and \(\mathbf{x}_{oo}\) correspond to the linear and angular velocities of the organ, respectively.

3) **Intraorgan Kinematics:** The relative linear and angular velocity of the EE with respect to a target point \(t_i\) on the inner surface of the organ (Fig. 8) is described as

\begin{align*}
\mathbf{v}_{gi}/t_i & = \begin{bmatrix} 0_{3 \times 3} \end{bmatrix} \mathbf{J}_h, \mathbf{\dot{q}}_h - \mathbf{x}_{ol} - \mathbf{T}_x \mathbf{x}_{oo} \\
\mathbf{n} & = \begin{bmatrix} \mathbf{I}_{3 \times 3} \end{bmatrix} \mathbf{J}_h, \mathbf{\dot{q}}_h - \mathbf{x}_{oo}. \quad (4)
\end{align*}

Combining (4) and (5) yields the twist of the EE relative to \(t_i\)

\begin{align*}
\mathbf{\dot{x}}_{gi}/t_i & = \mathbf{J}_h, \mathbf{\dot{q}}_h - \mathbf{H}_i \mathbf{\dot{x}}_o \quad (6)
\end{align*}

where

\begin{align*}
\mathbf{T}_i & = \begin{bmatrix} -\mathbf{a} \end{bmatrix} \\
\mathbf{H}_i & = \begin{bmatrix} \mathbf{I}_{3 \times 3} \mathbf{T}_i \mathbf{I}_{3 \times 3} \end{bmatrix}.
\end{align*}

4) **Euler Parameterization:** The mechanical structure of the hybrid robot in the organ cavity allows only 5 DoFs since independent rotation of the IODR EE about the \(\mathbf{x}\)–axis is unachievable due to the geometry of the organ constraint and the IODR. This rotation is represented by the third Euler angle \(\psi_i\) in a \(w\)–\(v\)–\(w\) sequence [41]. For the purposes of path planning and control, the twist of the EE can be parameterized using \(w\)–\(v\)–\(w\) Euler angles while eliminating the third Euler angle by using a degenerate matrix \(\mathbf{K}_i\) in (7). \(\mathbf{K}_i\) is defined in Appendix B. Inserting this new parameterization in (6) yields a relation between the achievable independent velocities and the joint parameters of the hybrid system as in (8) below:

\begin{align*}
\mathbf{\dot{x}}_{gi}/t_i & = \mathbf{K}_i \mathbf{\dot{q}}_h, \quad (7) \\
\mathbf{\dot{x}}_{gi}/t_i & = \mathbf{K}_i \mathbf{\dot{q}}_h. \quad (8)
\end{align*}

5) **Overall Organ–Robot System:** The robotic system must be constrained such that the multitude \(n_a\) of hybrid robotic arms moves synchronously to control the organ without tearing the insertion points. Let \(m_i\) designate the \(i\)th incision point on the organ, \(i = 1, 2, 3, \ldots, n_o\). Let \(m_i^\prime\) designate the corresponding point on the IODR cannula of the \(i\)th robotic arm that is instantaneously coincident with \(m_i\). To prevent damage to the anatomy, an equality constraint must be imposed between the projections of the linear velocities of \(m_i\) and \(m_i^\prime\) on a plane perpendicular to the longitudinal axis of the \(i\)th IODR cannula. These conditions are derived in Appendix C and given as

\begin{align*}
\mathbf{\dot{x}}_{Q_i} & = \mathbf{\dot{q}}_h (\mathbf{X}_{ol} + \mathbf{M}_i \mathbf{x}_{oo}), \quad i = 1, 2, 3, \ldots, n_o \quad (9) \\
\mathbf{\dot{x}}_{Q_i} & = \mathbf{\dot{q}}_h (\mathbf{X}_{ol} + \mathbf{M}_i \mathbf{x}_{oo}), \quad i = 1, 2, 3, \ldots, n_o. \quad (10)
\end{align*}

Equations (9) and (10) constitute \(2n_a\) scalar equations that provide the conditions for the organ to be constrained by \(n_a\) robotic arms inserted into it. For the organ to be fully constrained by the robotic arms, (9) and (10) should have a rank equal to the dimension of the organ twist \(\mathbf{x}_o\). If the organ is free-floating, then the rank should be six, and therefore, a minimum of three robotic arms is necessary to effectively stabilize the organ. If the organ is constrained from translation (e.g., the eye), the required rank is three, and hence, the minimal number of arms is two. This justifies the design of the dual-arm ophthalmic surgical system depicted in Fig. 1.

Combining (9) and (10) with the twist of the hybrid robotic arms \(\mathbf{\dot{x}}_{gi}/t_i\) for \(i = 1, 2, 3\) yields the kinematics of the overall system composed of the organ and the three robotic arms; see (11) below. Definitions of matrices \(G_i\) and \(P_i\) are given in Appendix C:

\begin{align*}
\mathbf{\dot{x}}_{Q_i} & = \mathbf{\dot{x}}_{Q_i} - \mathbf{\dot{x}}_{Q_i}, \quad (11)
\end{align*}
In Wen’s work [15], the kinematic and static model of a two-
finger grasping manipulator was given. By modeling the contact
between fingers and the payload mathematically, the authors
derived the differential kinematic relationship. Our application
differs because the insertion constraint produces a constantly
changing contact point on the robot arm. Equation (11) is ap-
plicable to general robotic systems with insertion constraints
piercing into hollow objects.

C. Jacobian Normalization

Robot kinetostatic performance can be evaluated by examin-
ing the characteristics of the robot Jacobian. Several commonly
used performance measures, like the condition number [23],
the manipulability index [19], and kinematic conditioning [42],
are based on the robot Jacobian. However, directly using the
Jacobian for performance evaluation leads to meaningless re-
sults due to the inhomogeneity of physical units of its compo-
nents [43]. Normalization of the Jacobian is necessary in order
to ensure meaningful results when calculating the singular val-
dues. Inhomogeneity of the units of the Jacobian stems from the
inhomogeneity of physical units of its compo-
nents in (14) while relating each matrix block to the
kinematics of the parallel robot, the IODR, or the organ.

The Jacobian matrix \( \mathbf{J}_O \) characterizes the velocities of the
rotating organ and the desired EE. \( \mathbf{J}_O \) is homogenized using the
radius of the organ at the target point as the characteristic length.
It is this radius, as measured with respect to the instantaneous
center of rotation, that imparts a linear velocity to point \( t_s \) as a
result of the angular velocity of the organ. The top right nine
components of \( \mathbf{K}_i \mathbf{H}_i \), \( i = 1, 2, \) and 3 of (11), bear the unit of
[m]. Hence, we divide them by the radius of the organ at the
target point, \( L_r \) in order to normalize them. This approach is
congruent with [14] and [44], which used scaling matrices re-
lying the units of twist of the EE. The same treatment is also
carried out to the rightmost six components of each matrix block
\( \mathbf{G}_i \mathbf{P}_i \), where we divide them by \( L_r \) as well.

Jacobian \( \mathbf{J}_I \) describes the geometry of both the parallel robot
and the IODR. Therefore, we use both \( L_p \), the length of the
connection link of the parallel robot (\( \frac{\rho_i}{Q_i} \) in Fig. 8), and \( L_r \), the
bending radius of the inner tube of the IODR as characteristic
lengths. We multiply \( L_p \) to those components in \( \mathbf{K}_i \mathbf{J}_i \) that bear
the unit of [1/m]. We also divide the components that bear the
unit of [m] by \( L_r \). This results in a normalized input Jacobian
\( \mathbf{J}_I \) that is dimensionless. While previous works suggested using
the radius of the moving platform for normalization [23], [27],
we chose \( L_p \) because we are interested in the angular and linear
velocities at point \( q_t \) (Fig. 8). \( L_p \) is the scaling factor of the
linear velocity at point \( q_t \) stemming from a unit angular velocity
of the moving platform. Similarly, the circular bending tube of
the IODR is modeled as a virtual rotary joint (Fig. 8); hence, the
bending radius \( L_s \) is used to normalize the components related
to the IODR.

V. MOTIVATING CASE STUDY: OPHTHALMIC MICROsurGERY

A. Model of the System in Ophthalmic Surgery

The eye is constrained by its surrounding musculature to have
only three rotational DoFs about its center. The twist of the organ
in (3) is reduced to a 3-D vector as in (15), shown below. The linear
and angular velocities of the robot arm EE relative to a
target point \( t_i \) on the retina are given by (16) and (17), shown below:

\[
\mathbf{\dot{x}}_r = [\dot{\alpha}, \dot{\beta}, \dot{\gamma}]^T
\]

\[
\mathbf{v}_{g_{i/t_i}} = [\mathbf{I}_{3\times3}, 0_{3\times3}] \mathbf{J}_i \mathbf{q}_i - \mathbf{T}, \mathbf{x}_r
\]

\[
\mathbf{\omega}_{g_{i/e_r}} = [0_{3\times3}, \mathbf{I}_{3\times3}] \mathbf{J}_i \mathbf{q}_i - \dot{\mathbf{x}}_r
\]

Combining (16) and (17) yields the relative twist of the EE of
each arm with respect to the target point on the retina (18), where
\( \mathbf{D}_i = [\mathbf{T}_i^T, \mathbf{I}_{3\times3}]^T \). The 5-D constrained twist of the IODR EE in
(8) simplifies to (19), shown below. The overall system Jacobian is given by (20), shown below:

\[
\ddot{x}_{p/t} = J_h \dot{q}_h - D \ddot{x}_e \\
\ddot{x}_{g/t} = J_h \dot{q}_h = K \ddot{x}_e
\]

\[
\begin{align*}
\begin{pmatrix}
\dot{q}_{h1} \\
\dot{q}_{h2}
\end{pmatrix} &=
\begin{pmatrix}
I_{5 \times 5} & 0_{5 \times 5} & K_{1} D_1 \\
0_{5 \times 5} & I_{3 \times 5} & K_{2} D_2 \\
G_1 F_1 & 0_{2 \times 8} & G_2 F_2 \\
G_1 F_1 & 0_{2 \times 8} & G_2 F_2
\end{pmatrix}
\begin{pmatrix}
\ddot{x}_{g_{1,t}} \\
\ddot{x}_{g_{2,t}}
\end{pmatrix} \\
& \quad \times
\begin{pmatrix}
N_1 \\
N_2
\end{pmatrix}
\end{align*}
\]

Rewriting (20) using matrices M and N, we obtain (21), shown below, where \( \dot{q}_h = [\dot{q}_{h1}^T, \dot{q}_{h2}^T]^T \), and \( \ddot{x}_{g/t} = [\ddot{x}_{g_{1,t}}, \ddot{x}_{g_{2,t}}]^T \). Note that intraocular manipulation can be modeled by setting \( \ddot{x}_e = 0 \). Similarly, organ manipulation can be represented by setting \( \ddot{x}_g/t = 0 \).

\[
M \dot{q}_h = N_1 \ddot{x}_{g/t} + N_2 \ddot{x}_e.
\]

B. Dexterity Evaluation for All Manipulation Cases

Fig. 4 showed four manipulation cases. The corresponding dexterity measures for these cases are derived in this section. Throughout this section, two incision points are specified by angles \([\theta, \phi]^T = [\pi/3, \pi/3]^T \) and \([\pi/3, \pi/3]^T \), as defined in Fig. 9.

1) Case 1: Intraocular Dexterity: The aim of intraocular dexterity evaluation is to compare the performance of the system with or without the IODR. The setup without the IODR (7-DoF setup) represents the current ophthalmic surgical setup based on rigid tools. For the setup without an IODR, a straight cannula capable of rotating about its own longitudinal axis was modeled, yielding a 7-DoF robotic arm. The Jacobian matrix for the 7-DoF robotic arm is

\[
J_{7\text{DoF}} = [B, A, J^{-1}_F, 0_{3 \times 1}, \dot{z}_Q, 0_{6 \times 1}/L_s] \begin{pmatrix}
I_{6 \times 6} \\
0_{6 \times 6} \\
1/L_s
\end{pmatrix}
\]

\[
J_{8\text{DoF}} = [B, A, J^{-1}_F, 0_{3 \times 1}, \dot{z}_Q, 0_{6 \times 2}/L_s] \begin{pmatrix}
I_{6 \times 6} \\
0_{6 \times 2} \\
1/I_s
\end{pmatrix}
\]

Based on clinical observation of vitrectomy procedures, we estimate that most procedures are performed on an area of \( \pm 20^\circ \) around a chosen center point on the retina. The lower hemisphere of the retina is divided into a number of \( \pm 20^\circ \) patches. These patches represent different possible surgical sites. The local KCI (LKCI) for each patch is calculated by averaging the KCI values over the scanned area and plotting the results throughout the workspace. In these plots, \( \theta \) and \( \phi \) are the first two \( w-v-w \) Euler angles that locate the center point of each patch (see Fig. 9).

Fig. 10 shows that the translational and rotational dexterities are significantly improved throughout the workspace by the use of the IODR. Table I also presents the global KCI (GKCI) defined as the average of LKCI values over all workspace patches.

Fig. 11 shows the robot’s intraocular tangential dexterity on a specified surgical site: \( 120^\circ < \theta < 160^\circ \) and \( 20^\circ < \phi < 100^\circ \). The manipulability ellipsoids of the robot while reaching each
site point have been intersected with the local tangent plane of the eye. The resulting ellipses represent the robot’s dexterity in performing tangential motions, such as tissue dissection. This information can be used for planning specific surgical tasks, e.g., bimanual membrane peeling.

2) Case 2: Orbital Dexterity With Constraints: This case is represented by Fig. 4(b). In this case, the two arms are constrained to maintain zero relative motion with respect to a target point on the retina while manipulating the eyeball. The target point was selected to be $[\frac{5\pi}{6}, 0]^T$, defined in the eye-attached coordinate system $\{E\}$. Frame $\{E\}$ is defined similarly as the organ coordinate system $\{O\}$ in Fig. 8, and represents the rotation of the eye relative to $\{W\}$.

To verify the accuracy of our derivation, we specified a desired rotation velocity of the eye as $10^\circ$ per second about $\hat{w}$-axis in Fig. 8, and calculated the joint velocities through the inverse of the Jacobian. For the task of rotating the eye by fixing the EE to the target point, the two IODRs and the eyeball form a rigid body allowing no relative motion in-between. Therefore, the rates of the IODR joints are expected to be zeros. Fig. 12 confirms the correctness of our derivation.

The robot dexterity in tilting the eye $20^\circ$ in all directions from its unperturbed “home” configuration was evaluated. The LKCI based on (26) was plotted in Fig. 13 using star symbols. Removing the constraints of zero relative velocity of the EE increases the orbital manipulation dexterity of the system.

4) Case 4: Simultaneous Orbital and Intraocular Dexterity: In case 4, both arms coordinate to manipulate the eyeball. One arm also operates inside the eye along a specified path. The overall dexterity of the robot in executing this combined motion is evaluated. The eye is rotated about $\hat{w}$-axis in Fig. 8, while one robotic arm scans the retina independently. An illustration of this combined motion is depicted in Fig. 14.

Fig. 15 shows the translational and rotational KCI values separately. Since we have put more constraints on the system (both orbital manipulation constraint and intraocular scanning constraint), the KCI values are smaller than for cases 1–3. In current practice, ophthalmic surgeons are not capable of performing simultaneous manipulations, as defined in case 4. We plan on using the robot to decrease the operation time and the number of delicate regrasping operations.
VI. CONCLUSION

This paper has presented a new hybrid robotic system being developed for surgical procedures associated with hollow suspended organs. Four simulation cases were proposed to define manipulation procedures for hollow organ surgeries. Specifically, the presented cases were examined in ophthalmic surgery where the eyeball is a partially constrained hollow suspended organ. Dexterity improvements in manipulation and operation of the eyeball were quantified while using the proposed robot. Our future research includes experimentally validating the robotic system shown in Fig. 1, which is currently under construction.

APPENDIX A

As shown in Fig. 8(a), in order to transform the linear and angular velocities from Stewart–Gough platform center to frame \(\{Q_i\}\), we have

\[
\begin{align*}
\mathbf{v}_{Q_i} &= \mathbf{v}_{P_i} + \omega_{P_i} \times (\mathbf{p}_{Q_i} - \mathbf{p}_{P_i}) \quad (27) \\
\omega_{Q_i} &= \omega_{P_i}.
\end{align*}
\]

Writing (27) and (28) in matrix form results in the twist of the distal end \(q_i\) of the connection link

\[
\dot{\mathbf{x}}_{Q_i} = \mathbf{A}_i \dot{\mathbf{x}}_{P_i}
\]

where \(\mathbf{A}_i = \mathbf{W}(\mathbf{r}_{P_i}q_i)\) is the twist transformation matrix defined in Section IV-A.

Similarly, we have \(\mathbf{B}_i = \mathbf{W}(\mathbf{q}_i\mathbf{m}_i)\) and \(\mathbf{C}_i = \mathbf{W}(\mathbf{m}_i\mathbf{q}_i)\) to calculate the twist of point \(g_i\) contributed by the Stewart–Gough platform. By incorporating the two serial DoFs of the IODR, we obtain the twist of point \(g_i\) as

\[
\dot{\mathbf{x}}_{G_i} = \mathbf{C}_i \mathbf{B}_i \dot{\mathbf{x}}_{Q_i} + \mathbf{C}_i \begin{bmatrix} 0 \\ \mathbf{z}_{Q_i} \end{bmatrix} \dot{q}_{s_{i1}} + \begin{bmatrix} \mathbf{r}_{G_i} \\ \mathbf{y}_{N_i} \end{bmatrix} \dot{q}_{s_{i2}}.
\]

Thus, we get the Jacobian \(\mathbf{J}_{s_{i}}\) of the IODR as

\[
\dot{\mathbf{x}}_{G_i} = \mathbf{C}_i \mathbf{B}_i \dot{\mathbf{x}}_{Q_i} + \mathbf{J}_{s_{i}} \dot{\mathbf{q}}_{s_{i}}
\]

where

\[
\mathbf{J}_{s_{i}} = \begin{bmatrix} \mathbf{H}_i & \mathbf{r}_{G_i} \\ \mathbf{y}_{N_i} \end{bmatrix}
\]

including the speeds of rotation about the axis of the IODR cannula and the bending of the precurved NiTi tube. The hybrid Jacobian matrix relating the twist of point \(g_i\) and all eight inputs of one arm was obtained as in (2), where \(\mathbf{J}_{h_i} = [\mathbf{C}_i \mathbf{B}_i \mathbf{A}_i \mathbf{J}^{-1}_{P_i} \mathbf{J}_{s_{i}}]\), and \(\dot{\mathbf{q}}_{h_i} = \begin{bmatrix} \dot{\mathbf{q}}_{P_{i1}} \\ \dot{\mathbf{q}}_{s_{i}} \end{bmatrix}^T\).

APPENDIX B

The 5 \times 1 Euler angle parameterization of the desired \(i\)th EE velocity \(\dot{\mathbf{x}}_{S_{i}}/t_i\) is related in the text to the general twist of the \(i\)th robot EE \(\dot{\mathbf{x}}_{S_{i}}/t_i\), by the degenerate matrix \(\mathbf{K}_i\). The matrix is derived using the relationship found in the literature [45] relating the Cartesian angular velocities to the Euler angle velocities

\[
[\omega_x, \omega_y, \omega_z]^T = \mathbf{R}_i \begin{bmatrix} \phi \dot{\phi} \theta \dot{\theta} \end{bmatrix}^T
\]

(32)

where

\[
\mathbf{R}_i = \begin{bmatrix} 0 & -\sin(\phi_i) & \cos(\phi_i) \sin(\theta_i) \\ 0 & \cos(\phi_i) & \sin(\phi_i) \sin(\theta_i) \\ 1 & 0 & \cos(\theta_i) \end{bmatrix}.
\]

With the aforementioned relationship, the general twist of a system \(\dot{\mathbf{x}}\) can be related to the 6 \times 1 Euler angle twist \([\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\phi}]^T\) as follows:

\[
[\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\phi}]^T = \mathbf{S}_i \dot{\mathbf{x}}
\]

(33)

where

\[
\mathbf{S}_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathbf{R}_i^{-1} \end{bmatrix}.
\]

The 5 \times 1 Euler parameterization used in the path planning equation of the text is derived by applying a 5 \times 6 degenerate matrix to the 6 \times 1 Euler angle twist as follows:

\[
\tilde{\mathbf{x}} = [I_{5 \times 5}, 0_{5 \times 1}] [\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\phi}]^T.
\]

(34)

Substituting the relationship between the generalized and the 6 \times 1 Euler angle twist (33) yields the matrix \(\mathbf{K}_i\) as follows:

\[
\tilde{\mathbf{x}} = \mathbf{K}_i \dot{\mathbf{x}} \quad \text{where} \quad \mathbf{K}_i = [I_{5 \times 5}, 0_{5 \times 1}] \mathbf{S}_i.
\]

APPENDIX C

In the text, we specified the constraint that each insertion arm moves at the insertion point only with the velocity equal to the velocity of the organ surface at that point, plus any velocity along the insertion needle. To assist in the development of this constraint, point \(m_i\) is defined at the insertion point on the surface of the organ and \(m_i'\) is defined as the point on the insertion needle instantaneously coincident with \(m_i\) (Fig. 8). The velocity of \(m_i'\) must be equal to the velocity of point \(m_i\) in the plane perpendicular to the needle axis

\[
\mathbf{v}_{m_i'} = \mathbf{v}_{m_i, \perp}.
\]

(36)
Taking a dot product in the directions $\hat{x}_{Q_i}$ and $\hat{y}_{Q_i}$ yields two independent constraint equations

$$\hat{x}_{Q_i}^T \dot{v}_{m_i} = \hat{x}_{Q_i}^T \dot{v}_{m_i},$$

(37)

$$\hat{y}_{Q_i}^T \dot{v}_{m_i} = \hat{y}_{Q_i}^T \dot{v}_{m_i}.$$  

(38)

These constraints can be expressed in terms of the joint angles and organ velocity by relating the velocities of point $m_i$ to the robot and organ coordinate systems. The velocity of point $m_i'$ can be related to the velocity of frame $\{Q_i\}$ as

$$\dot{v}_{m_i'} = v_{Q_i} + \omega_{Q_i} \times \hat{q}_m,$$

(39)

By substituting the twist of frame $\{Q_i\}$ (39) becomes

$$\dot{v}_{m_i'} = \begin{bmatrix} I_{3x3} & 0_{3x3} \end{bmatrix} \hat{x}_{Q_i} + E_i [0_{3x3}, I_{3x3}] \hat{x}_{Q_i},$$

(40)

where

$$E_i = \begin{pmatrix} -\hat{q}_i \hat{m}_i \end{pmatrix}. $$

Inserting (29) and (1) and writing in terms of the hybrid joint parameters $\dot{q}_b_i$ yields

$$\dot{v}_{m_i'} = F_i \dot{q}_b_i,$$

(41)

where

$$F_i = \begin{bmatrix} (I_{3x3}, 0_{3x3}) + E_i [0_{3x3}, I_{3x3}] \end{bmatrix} A_i J_i^{-1} [I_{6x6}, 0_{6x2}].$$

An expression for the velocity of the insertion point $m_i$ can be related to the desired organ velocity, yielding

$$\dot{v}_{m_i} = \hat{x}_{so} + M_i \hat{x}_{oa},$$

(42)

where

$$M_i = \begin{pmatrix} -\hat{m}_i \end{pmatrix}. $$

Substituting (41) and (42) into (37) and (38) yields the final constraint equations given the rigid-body motion of the organ–robot system

$$\hat{x}_{Q_i} F_i \dot{q}_b_i = \hat{x}_{Q_i} (\hat{x}_{so} + M_i \hat{x}_{oa}),$$

(43)

$$\hat{y}_{Q_i} F_i \dot{q}_b_i = \hat{y}_{Q_i} (\hat{x}_{so} + M_i \hat{x}_{oa}).$$

(44)

For convenience, vectors $\hat{x}_{Q_i}$ and $\hat{y}_{Q_i}$ are put in matrix form as $G_i = [\hat{x}_{Q_i}, \hat{y}_{Q_i}]^T$, and we define $P_i = [I_{3x3}, M_i]$. As a reminder, these matrices are used to derive the overall Jacobian equation, as shown in (11).

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REFERENCES


Wei Wei (S’06) received the B.Eng. degree from Tsinghua University, Beijing, China, in 2003, the M.Eng. degree from McGill University, Montreal, QC, Canada, in 2005, and the M.Phil. degree from Columbia University, New York, NY, in 2007, all in mechanical engineering. He is currently working toward the Ph.D. degree with the Department of Mechanical Engineering, Columbia University. His current research interests include parallel robots, flexible robots, medical robotics, mechanical system synthesis, and control systems.

Roger E. Goldman received the B.Sc. degree in mechanical engineering from Stanford University, Stanford, CA, in 2002. He is currently working toward the M.D. and Ph.D. degrees in biomedical engineering at Columbia University, New York, NY. In 2003, he joined Foxhollow Technologies, Inc., Redwood City, CA, where he developed catheters for endovascular procedures. He is currently a National Institutes of Health (NIH) Medical Scientist Training Program Fellow at Columbia University. His current research interests include novel robotic instruments for applications in medical diagnosis and therapy.

Howard F. Fine received the B.S. degree in chemical engineering and biology from the Massachusetts Institute of Technology, Cambridge, in 1996, the M.D. (cum laude) degree from Harvard Medical School, Boston, MA, in 2001, and the M.H.Sc. degree in clinical trials from Duke University Medical Center, Durham, NC, in 2000.

In 2005, he completed the Ophthalmology Residency at Wilmer Eye Institute, Johns Hopkins Medical Institutes, Baltimore, MD, and, in 2007, the fellowship training in vitreoretinal surgery from Edward S. Harkness Eye Institute, Columbia University Medical Center, New York, NY, where he is currently the Medical Director of the Gerstner Clinical Research Center and an Assistant Professor of ophthalmology. He has authored or coauthored literature on minimally invasive vitreoretinal surgical outcomes, autofluorescence imaging of the retina, and the use of monoclonal antibodies in the treatment of retinal vascular diseases. His current research interests include expanding vitreoretinal surgical capabilities, retinal imaging, and novel therapeutics for retinal vascular disease.

Nabil Simaan (M’03) received the B.Sc., M.Sc., and Ph.D. degrees in mechanical engineering from Technion—Israel Institute of Technology, Haifa, Israel, in 1996, 1999, and 2002, respectively. His Masters and Ph.D. research focused on the design, synthesis, and singularity analysis of parallel robots for medical applications, stiffness synthesis, and modulation for parallel robots with actuation and kinematic redundancies. In 2003, he was a Postdoctoral Research Scientist at Johns Hopkins University National Science Foundation (NSF) Engineering Research Center for Computer-Integrated Surgical Systems and Technology (ERC-CISST), Baltimore, MD, where he focused on minimally invasive robotic assistance in confined spaces. In 2005, he joined Columbia University, New York, NY, as an Assistant Professor of mechanical engineering and the Director of the Advanced Robotics and Mechanisms Applications (ARMA) Laboratory. During the course of his research, he designed and constructed compact parallel robots for medical applications and compact and downsizable surgical slaves for minimally invasive surgery of the throat and upper airway. His current research interests include synthesis of novel robotic systems for surgical assistance in confined spaces with applications to minimally invasive surgery of the throat, natural orifice surgery, cochlear implant surgery, and dexterous bimanual microsurgery.