ME8331 Robot Manipulators Term Projects

Contents

1 Requirements

<u>Due date</u>: 4/28/2014 (presentation) and 4/30/2014 (report)

On 4/28/14 you will present a 15 minute presentation followed up by 5 minutes of questions about your work. During this presentation you will focus on explaining the problem and how you mathematically modeled it and verified the theory using simulations. The focus of the presentation will be on the breadth of literature review that your present, the thoroughness of your results, the success in conveying your lessons learned from your project to your class members and your ability to answer questions. Undergraduate students are allowed to team up in teams consisting no more than 2 students. In this case the presentation must be divided so as to allow each student to present roughly half of the slides and to answer half of the questions. Graduate students are required to submit individual projects.

On 4/30/2014 you will be required to submit a written report including all programs that you wrote and your PowerPoint presentation slides (please provide the original *.ppt or *.pptx files). In the following list of project topics you will notice a number enclosed with brackets (e.g. [$*x.0$]). This number designates the *Difficulty Factor* of the project as a general guide to assist you in choosing the project topics. <u>Here are the rules and guidelines about how the difficulty factor will be used:</u>

(a) The final score of your project will be the product of original graded score and the difficulty factor.

(b) If you are an undergraduate student and you are working alone, then your difficulty factor will be added by 0.1; If you team up with another undergraduate, your team will be seen as ”one graduate student”. Your presentation grade may be different based on how well you individually answer questions and present your slides.

(c) Projects with difficulty score of $\leq$ [$*1.0$] are expected to be fully completed with all aspects of project requirements. Projects with difficulty score $\geq$ [$*1.0$] have typically some potential pitfalls or open-ended questions or a component of guided self-study significantly beyond the material provided in class. To mitigate the risk involved in a student getting stuck with one or more aspects of the project we use the difficulty factor.

(d) A difficulty factor [*0.9] project is usually good for undergraduate student projects. There will be less implementation coding and theoretical study work involved, instead more design and performance metric evaluation work.

(e) A difficulty factor [*1.0] project is a good project for graduate student or a team of 2 undergraduate students. The students are expected to fully complete all the specifications given in the problem.

(f) A difficulty factor [*1.1] project will involve more implementation coding and more system complexity than [*1.0].

(g) A difficulty factor [*1.15 - 1.2] project will involve heavy implementation coding and high system complexity, or it has aspects that are open-ended or require guided self-study.

<u>A note about academic integrity and code/project dissemination:</u> While you are encouraged to show your projects to future employers and colleagues please do not post your code and projects to online web-sites such as GitHub or other social/code sharing sites. Do not reuse any component of coding of reports from previous years - even if you accidentally are granted access to the information. Failure to follow these guidelines will result in an invitation to the student to appear in front the student honor council for academic integrity.

2 Kinematic Redundancy Resolution

The followings are suggested topics related to redundancy resolution.

2.1 Application of global optimization methods for redundancy resolution [*1.2]

Based on [22, 23], summarize the algorithm. Explain the relationship between the algorithm and variational calculus. Simulate the same robot as in [23] for optimizing obstacle avoidance and joint rates. In your presentation explain the difference between this method and the methods learned in class. Explain the theoretical background behind the method, and present your simulations.

2.2 Local/Global dexterity optimization using local redundancy resolution [*0.9]

Assume you have a Puma 560 robot carrying a small 6 DoF Stewart/Gough Platform. The specific dimensions of the robots will be provided by the instructor.

(a) Given joint errors for the Puma and the parallel robot, simulate the error ellipsoids for each robot at sample poses in their respective workspace.

(b) Simulate the combined error ellipsoid for the hybrid robot.

(c) Propose and implement a redundancy resolution that maximizes the precision of assembly tasks while respecting joint limits and maximizing dexterity.

(d) Simulate your redundancy resolution along a chosen assembly task and compare the performance to pseudo-inverse resolved rates for the hybrid robot.

2.3 Hybrid robot redundancy resolution [*0.9]

Assume you have a hybrid robot composed from n parallel robots (Stewart/Gough platform) that are stacked in series. The segmented line connecting the centers of each base and moving platform of each parallel robot will be called the backbone of the robot.

(a) Create a kinematic modeling framework by treating the backbone as a serial robot where each line segment of the backbone is treated as an imaginary prismatic joint that can tilt in 2 DoF with respect to the preceding line segment.

(b) Derive the instantaneous kinematics Jacobian for the imaginary backbone model assuming $n = 3$.

(c) Define a redundancy resolution that solves for the imaginary joint rates of this backbone.

(d) Use your results in c to derive the joint speeds of each parallel robot

(e) Create a kinematic simulation of this robot and calculate the Jacobian of the hybrid robot for $n = 3$.

2.4 Redundancy resolutions for operating 6 DoF robots [*1.0]

Assume you have a 6 DoF robot. Assume your robot is reaching through a constraint tunnel in order to reach its target operation site. Your goal is to carry out a range of tasks requiring 3, 4, and 5 DoF while avoiding obstacle constraints imposed by the access tunnel.

(a) Define an optimization framework that allows you to cast the problem as a redundancy resolution problem. Refer to the papers [3, 8, 20] for inspiration.

(b) Simulate a robot of your choice for carrying out 3 DoF tasks

(c) Simulate a robot of your choice for carrying out 5 DoF tasks

(d) Compare two solution alternatives and draw conclusions.

2.5 Obstacle avoidance redundancy resolution [*1.0]

Study the works of [8, 20, 23] and create a simulation of a serial robot reaching into a confined space.

(a) Discuss with the instructor the choice of robot and constrained space. Define the D-H parameters of your robot and derive its kinematics and Jacobian.

(b) For each point along the length of the robot define a jacobian defining the imparted velocity of that point as a function of joint speeds

(c) Define at least two methods for simulating obstacle avoidance either using potential functions or using task priority redundancy resolution method

(d) Simulate both methods and compare the ease of determining the tuning parameters to obtaining good results

3 Robot Statics & Actuation Redundancy Resolution

3.1 Actuation redundancy resolution for a parallel robot [*1.0]

Assume you have a planar parallel robot of the type 4RPR (4 kinematic chains with, revolute, prismatic, and revolute joints). The only active joints are the prismatic joints.

(a) Formulate the kinematics jacobian of this robot

(b) Formulate the passive and active stiffness of this robot.

(c) Assume your goal is to minimize the load experienced on the legs and to make the load distribution as even as possible. Define a redundancy resolution strategy and simulate the performance of the robot for a given load at its tip .

(d) Assume now that one of the actuators has reduced load carrying capabilities that is 50% compared to the other actuators. Define a redundancy resolution strategy and simulate the robot performance in performing the same task as in (c).

(e) Simulate the load carrying capabilities of the robot throughout its workspace while assuming the moving platform is kept at fixed orientation.

(f) Compare your results in (e) and (c). Present plots quantifying the change in attainable workspace while not violating joint force limits.

3.2 Static Balancing of Parallel Mechanisms and Robots [*1.2]

Given a 3RRR parallel robot, generate a formulation methodology for solving the static balancing for this robot.

(a) Conduct a literature review on static balncing of parallel robots

(b) Assuming that you can place torsional springs in all revolute joints, formulate the statics and kinematics of the parallel robot

(c) Formulate the expression for the elastic and gravitational energy of the robot

(d) Cast the problem of static balancing as an optimization problem

(e) Solve the problem using numerical methods.

(f) Assume you have 3, 6, or 9 springs and compare the complexity and the robustness of the static balancing to parameter perturbation in spring constants.

(g) Submit a simulation that calculates a quality metric for the static balancing within the workspace of the robot

3.3 Stiffness Modulation of Pneumatic Parallel Robots [*1.2]

Assume you are using pneumatic actuators (e.g. piston actuators or McKibben artificial muscles) to construct an MRI-compatible parallel robot. Assume that your robot is not accurately registered to the anatomy of a patient and that motions due to breathing of the patients cause the anatomy to move up/down by $\pm 5mm$.

(a) Conduct a literature review on pneumatically actuated parallel robots and stiffness modulation using antagonistic actuation or actuation redundancy

(b) Consider a modified geometry of the Delta robot (e.g. [19]) that allows you to use the pneumatic actuator technology. Present the design and mobility analysis and discuss how you achieve antagonistic actuation

(c) Formulate the kinematics and statics for this robot and derive an expression for the Jacobian of the robot

(d) Assume joint-level stiffness controlled by your low-level pneumatic control. Derive the robot stiffness while considering the passive and active stiffness

(e) Derive an actuation redundancy resolution and simulate the problem of stiffness modulation using antagonistic actuation/actuation redundancy. See [17] and [16] for the terminology for actuation redundancy. Discuss the actuation redundancy with the instructor when planning your simulations.

(f) Simulate the robot within an x-y cross section of its workspace and assume that the robot tip is perturbed $\pm 5mm$ in the z direction. Calculate the changes in the reaction force due to this perturbation.

(g) Simulate the extent of stiffness modulation you can get assuming you may modify the joint-level stiffness by $\pm 40\%$ and that you can also use the active stiffness component. Compare the contribution of the active stiffness modulation to the overall stiffness v.s. the passive stiffness modulation by controlling stiffness at joint space.

4 Robot Dynamics and Control

4.1 Computed Torque (PD+Inverse Dynamics) control [*1.1]

Implement a simulation of a PD+Inverse dynamics of a Puma 560 robot.

(a) Develop the dynamics model of the robot using a Largangian approach or (strongly suggested) use a dynamic model to be provided by the instructors.

(b) Simulate the step response (in joint space) of the controller and investigate the effect of the P and D gains.

(c) Assume the robot carries a 2Kg mass at its tip but the controller does not know this information. Simulate the robot following a quintic polynomial linear trajectory and compare the tracking error with this uncertainty.

(d) Propose and develop a robust min-max controller and compare its performance to section (c).

(e) Compare the results of the min-max controller and the naive computed torque controller based on the nominal dynamics. Present plots of tracking errors in task space and in joint space for both controllers. The plots will present tracking errors as a function of a parametric variable $t \in [0, 1]$ designating the location between the start point and the final point along the trajectory.

4.2 Computed Torque (PD+Inverse Dynamics) control [*1.1]

Implement a simulation of a PD+Inverse dynamics of a Fanuc ArcMate robot (specifications will be given by the instructors).

(a) Develop the dynamics model of the robot using a Largangian approach while focusing on capturing the dynamics of the first three joints.

(b) Simulate the step response (in joint space) of the controller and investigate the effect of the P and D gains.

(c) Assume the robot carries a 2Kg mass at its tip but the controller does not know this information. Simulate the robot following a quintic polynomial linear trajectory and compare the tracking error with this uncertainty.

(d) Propose and develop a robust min-max controller and compare its performance to section (c).

(e) Compare the results of the min-max controller and the naive computed torque controller based on the nominal dynamics. Present plots of tracking errors in task space and in joint space for both controllers. The plots will present tracking errors as a function of a parametric variable $t \in [0, 1]$ designating the location between the start point and the final point along the trajectory.

4.3 Hybrid motion/force control [*1.2]

Given a PD+inverse dynamics model of a Puma 560 robot develop a hybrid force/motion control simulation using Simulink.

(a) Summarize the literature review on hybrid force motion control and methods for decoupling the control action of the force and motion controller.

(b) Specify the selection/projection matrix for decoupling the controller tasks.

(c) Simulate the task of cleaning a whiteboard and quantify the performance of the robot.

(d) Simulate the task of inserting a peg in a hole.

(e) Simulate the task of turning a crank.

4.4 Hybrid motion/impedance control [*1.1]

Given a PD+inverse dynamics model of a Cartesian 3 DoF robot develop a hybrid impedance/motion control simulation using Simulink.

(a) Summarize the literature on impedance control

(b) Simulate the task of cleaning a whiteboard and quantify the performance of the robot.

(c) Test the performance of the controller to uncertainty in the stiffness model of the environment

4.5 Detection and Control for Magnetic Coupling Recovery of a Capsule [*1.1]

Assume you have a robot dragging a capsule within a tunnel characterized by a known viscous and Coulomb friction. The external magnet pulls the capsule via magnetic coupling which, for the purpose of this project, is modeled as an "artificial rubber" having the coupling force inversely proportional to the third power of the distance between the robot tip holding an external magnet and the capsule (i.e. $f = \frac{c}{d^3}$ where d is the distance and c is a constant for this simulation).

(a) Review the literature of hybrid admittance/motion control.

(b) Propose an algorithm for detecting when a magnetic coupling is being challenged by a capsule stuck inside a pocket in the tunnel

(c) Devise an algorithm for hybrid admittance/motion control in order to safely recover the capsule from an open pocket it is stuck in

(d) Write a Matlab code assuming you are using a 6 degrees of freedom robot and simulate the algorithm for recovering a stuck magnetic capsule

(e) Assume that your force measurements are corrupt with noise. Simulate the effect of this noise on your algorithm.

5 Robot Calibration & Visual Servoing

5.1 Application of kinematics methods for hand-eye calibration [*1.1]

Assume you have a 6 DoF Fanuc Arc-Mate robot with given kinematic model. Assume that you create an artificial model for an ideal camera that tells you the relative position of a fixed object in space with respect to a coordinate frame of the camera. Assume that the camera is fixed on the robot. Your aim is to calculate the position and orientation of the camera with respect to the robot end effector. In your solution you will create multiple vision points using the direct kinematics and then use the algorithm of [2]. Your presentation and report should include:

(a) A detailed summary of at least 2 different methods widely used in the literature

(b) Details of your simulation

(c) Plots of calibration convergence

(d) Assume now that all your camera points have a random noise of a maximal norm of 1 mm in position and 1 degree in orientation. Run your code again and see what your calibration results be as a function of the number of points used for calibration.

5.2 Calibration of Puma 560 robot [*1.15]

Define a framework for calibrating a Puma 560 robot assuming you have joint value information and end-effector information but the specific home position of the robot is not exact.

(a) Define calibration-robust parameters and explain why DH parameters are problematic

(b) Define the calibration Jacobian and write a simulation demonstrating recursive-least squares calibration.

(c) Given a set of real data points using the Puma 560 robot obtain the calibration parameters of the robot and compare the robot accuracy before and after calibration.

(d) Explain the calibration sensitivity index and simulate it throughout the workspace of the robot.

5.3 Hand-eye calibration of ultrasound probes [*1.15]

Assume you have an ultrasound probe with calibrated intrinsic parameters accounting for image deformation in the imaging plane. Assume that this probe is fixed to a serial robot with given kinematics. Your task is to simulate and to demonstrate the task of hand eye calibration of the ultrasound probe.

(a) Study the works [7, 35, 41], and summarize the problem of hand eye calibration.

(b) Summarize the key literature on ultrasound hand-eye calibration and explain the differences between hand-eye calibration of cameras and Ultrasound probes.

(c) Summarize the work on ultrasound calibration phantom as related to hand-eye calibration. Create a Matlab code that calculates the intersection points between a given plane and a collection of lines in space. Consult [30] for efficient calculation of these intersections.

(d) Create a Matlab simulation assuming the US probe is mounted on a Puma 560 robot. Initially assume the US probe is mounted in a known pose relative to the robot and create a data set called "measurements" that will be used for calibrating the probe location.

(e) Create a Matlab code that simulates your approach based on the literature you read on hand-eye claibration of US probes. Use the "measurements" data set to run your calibration probe.

(f) Characterize the calibration error if you contaminate the measurements data with 5%-10% random error noise.

5.4 Eye-In-Hand Visual Servoing Simulation [*1.15]

Assume you have a PUMA 560 and a camera mounted at the end-effector. Assume the position and orientation of the camera with respect to the gripper is known.

(a) Model the direct kinematics of the robot

(b) Use the matlab commands campos, camtarget, and camup to simulate the view from the camera attached to the gripper.

(c) Derive the Jacobian of the end-effector in world frame.

(d) Derive the Camera Jacobian in end-effector frame.

(e) Derive the Camera Jacobian in world frame.

(f) Write the resolved rate algorithm in camera frame, (your error will be a 2D vector in camera frame) and show that the robot+camera is able to follow a line in X direction, a line Y in the direction, and a circle in camera frame. Keep orientation fixed. You will show both the view from the camera and the overall view of the robot.

(g) Use standard color segmentation algorithms to track a point in camera view. Assume the point moving in a plane parallel to the camera plane. Move the point with one of the trajectories you already used in the course (i.e. from HW2 or HW3).

(h) Use visual servoing algorithms to keep the point in the center of the camera view. Record movies from both camera view and a general view.

6 Robot Design & Analysis

6.1 Stability of wire-actuated parallel robot with 8 wires [*1.1]

Evaluate the effectiveness of actuation redundancy in enlarging the workspace of the robot and in providing stiffness modulation. This work will be based on [36, 38, 39] and other works provided by the instructor. Evaluate the improvement in dexterity and global kinematic conditioning. The simulation will be based on an 8-wire actuated parallel robot to be specified by the instructor.

In the presentation you should focus on: 1) Explain the conditions for stability of wire actuated robots. 2) explain the relationship between the number of wires and the mobility of the moving platform has 3) explain the mathematical algorithm for solving the statics and the kinematics of a general 6 DoF wire actuated robot with n wires (where n¿6). 4) explain the concept of dexterity of wire actuated robots 5) explain the problem of stiffness modulation as defined in [36, 38, 39]. 6) Propose and explain the algorithm for computing the reachable workspace of wire actuated robots.

In the written report you should include all material included in the presentation and also include Matlab simulations that show 1) plot the workspace of the wire actuated robots in 3D for both the configurations with 7 and 8 wires. 2) Calculate the % increase in the work volume as the number of actuation wires is increased from 7 to 8 wires while maintaining the same maximal allowable axial strain in the wires. 3) generate condition number plots through an x-y cross section of the workspace 4) Calculate the improvement in the global conditioning index over the reachable workspace while assuming the moving platform remains parallel to the base.

6.2 Application of computational line geometry for singularity detection [*1.1]

Based on the works of [30, 31, 37, 43]. The work should mainly be based on [30] and you should scan the workspace of a Stewart 6-6 robot searching for singularity and then at a given singular configuration you should perform the singular complex approximation to obtain the pitch and axis of the twist associated with the self motion of the robot and then you should explain the expected behavior of the robot. Specifically, you are required to explain the Fichter and Hunt singularities described in [10, 14] by using the singular complex approximation. You will be required to write a Matlab simulation that shows the robot at a singular configuration and also presents the results obtained from the singular complex approximation.

In the presentation you should explain the mathematical modeling of singular complex approximations including the singular value decomposition and the generalized eigenvalues problems (proof that the problem amounts to a generalized eigenvalues problem). Explain some of the results presented in the literature.

In the written report you should summarize and include all the proofs related to the singular values approximation algorithm presented in [30, 31, 37, 43]. You should also include the Matlab simulation and the results related to explaining the Fichter and Hunt singularities described in [10, 14] by using the singular complex approximation.

6.3 Design and quantification of performance of a parallel wrist [*0.9]

Assume you have a 3 DoF parallel kinematics wrist (specific type should be discussed with the instructor).

(a) Write the kinematics of the parallel wrist.

(b) Simulate the orientation workspace of the robot

(c) Quantify the orientation dexterity of the robot including KCI.

(d) Optimize the dimensions of the robot to maximize orientation dexterity.

(e) Explore the literature for use of quaternions for performance quantification of such a mechanism and apply at least one method using quaternions.

6.4 Stability of grasps with soft fingers having point contact with friction [*1.0]

Based on extending the material presented in class and also based on the works of [6, 15, 21, 34]. The problem will be to analyze the stability of a 3-fingered hand holding a ball with three fingers. During the presentation you should:

(a) Explain the mathematical modeling of the problem

(b) Explain the conditions for stability assuming linearized finger model

(c) Explain the difference between linearized and non-linearized finger models

(d) Explain a kinematic model of a ball held by 3 fingers and perturbed by two angles θ_x and θ_y (I suggest using quaternions). This kinematic model should relate the movement of the contact points on the ball with the movement along the finger spring.

(e) Explain concept of grasp regions as defined by friction cones.

(f) Explain the relationship with line geometry as explained by [29].

For the written report also include in addition to the problem formulation the following additional material:

(a) Write a simulation and determine the stability bounds on the preload of the springs modeling the fingers.

(b) Calculate the determinant of the Hessian of the grasp.

(c) Calculate the grasp forces using the grip matrix.

References

[1] Eugene L Allgower and Kurt Georg. *Introduction to numerical continuation methods*, volume 45. SIAM, 2003.

[2] Jorge Angeles, Gilbert Soucy, and Frank P Ferrie. The online solution of the hand-eye problem. *Robotics and Automation, IEEE Transactions on*, 16(6):720–731, 2000.

[3] Andrea Bajo, Ryan B Pickens, S Duke Herrell, and Nabil Simaan. Constrained motion control of multisegment continuum robots for transurethral bladder resection and surveillance. In *Robotics and Automation (ICRA), 2013 IEEE International Conference on*, pages 5837–5842. IEEE, 2013.

[4] Ronen Ben-Horin, Moshe Shoham, and Shlomo Djerassi. Kinematics, dynamics and construction of a planarly actuated parallel robot. *Robotics and Computer-Integrated Manufacturing*, 14(2):163–172, 1998.

[5] Ronen Ben-Horin, Moshe Shoham, and Shlomo Djerassi. Task-based design and synthesis of variable geometry parallel robots. *Mechanical Engineering*, Ph.D, 2002.

[6] V Brodsky and M Shoham. On the modeling of grasps with a multi-fingered hand. In *Computational Kinematics 95*, pages 271–280. Springer, 1995.

[7] Alexis Cheng, Martin K. Ackerman, Gregory S. Chirikjian, and Emad M. Boctor. Design and development of an ultrasound calibration phantom and system. *Proc. SPIE 9036, Medical Imaging 2014: Image-Guided Procedures, Robotic Interventions, and Modeling*, 9036:903624, March 2014.

[8] Gregory S Chirikjian and Joel W Burdick. An obstacle avoidance algorithm for hyper-redundant manipulators. In *Robotics and Automation, 1990. Proceedings., 1990 IEEE International Conference on*, pages 625–631. IEEE, 1990.

[9] Peter A Cummings. Continuation methods for qualitative analysis of aircraft dynamics. 2004.

[10] Eugene F Fichter. A stewart platform-based manipulator: general theory and practical construction. *The International Journal of Robotics Research*, 5(2):157–182, 1986.

[11] Ferdinand Freudenstein and Bernhard Roth. Numerical solution of systems of nonlinear equations. *Journal of the ACM (JACM)*, 10(4):550–556, 1963.

[12] Masoud Ghazvini. Reducing the inverse kinematics of manipulators to the solution of a generalized eigenproblem. In *Computational Kinematics*, pages 15–26. Springer, 1993.

[13] Clément M Gosselin. Static balancing of spherical 3-dof parallel mechanisms and manipulators. *The International Journal of Robotics Research*, 18(8):819–829, 1999.

[14] KH Hunt. Structural kinematics of in-parallel-actuated robot-arms. *Journal of Mechanical Design*, 105(4):705–712, 1983.

[15] Jeffrey Kerr and Bernard Roth. Analysis of multifingered hands. *The International Journal of Robotics Research*, 4(4):3–17, 1986.

[16] Sungbok Kim Sungbok Kim. Operational quality analysis of parallel manipulators with actuation redundancy. In *Proceedings of International Conference on Robotics and Automation*, volume 3, 1997.

[17] S. Kock and W. Schumacher. A parallel x-y manipulator with actuation redundancy for high-speed and active-stiffness applications. In *Proceedings. 1998 IEEE International Conference on Robotics and Automation (Cat. No.98CH36146)*, volume 3, 1998.

[18] Thierry Laliberté, Clément M Gosselin, and Martin Jean. Static balancing of 3-dof planar parallel mechanisms. *Mechatronics, IEEE/ASME Transactions on*, 4(4):363–377, 1999.

[19] M López, E Castillo, G García, and A Bashir. Delta robot: inverse, direct, and intermediate Jacobians. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 220:103–109, 2006.

[20] Anthony A Maciejewski and Charles A Klein. Obstacle avoidance for kinematically redundant manipulators in dynamically varying environments. *The international journal of robotics research*, 4(3):109–117, 1985.

[21] Matthew T Mason and J Kenneth Salisbury Jr. Robot hands and the mechanics of manipulation. 1985.

[22] Yoshihiko Nakamura. *Advanced robotics: redundancy and optimization*. Addison-Wesley Longman Publishing Co., Inc., 1990.

[23] Yoshihiko Nakamura and Hideo Hanafusa. Optimal redundancy control of robot manipulators. *The International Journal of Robotics Research*, 6(1):32–42, 1987.

[24] Alba Perez and J Michael McCarthy. Dimensional synthesis of bennett linkages. *Journal of Mechanical Design*, 125(1):98–104, 2003.

[25] Alba Perez and J Michael McCarthy. Dual quaternion synthesis of constrained robotic systems. *Journal of Mechanical Design*, 126(3):425–435, 2004.

[26] Alba Perez and J Michael McCarthy. Clifford algebra exponentials and planar linkage synthesis equations. *Journal of Mechanical Design*, 127(5):931–940, 2005.

[27] Alba Perez-Gracia and J Michael McCarthy. Kinematic synthesis of spatial serial chains using clifford algebra exponentials. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 220(7):953–968, 2006.

[28] RP Podhorodeski, AA Goldenberg, and RG Fenton. A null-space solution of the inverse kinematics of redundant manipulators based on a decomposition of screws. *Journal of Mechanical Design*, 115(3):530–539, 1993.

[29] Jean Ponce, Steve Sullivan, Attawith Sudsang, Jean-Daniel Boissonnat, and Jean-Pierre Merlet. On computing four-finger equilibrium and force-closure grasps of polyhedral objects. *The International Journal of Robotics Research*, 16(1):11–35, 1997.

[30] Helmut Pottmann, Martin Peternell, and Bahram Ravani. An introduction to line geometry with applications. *Computer-aided design*, 31(1):3–16, 1999.

[31] Helmut Pottmann and Johannes Wallner. Approximation in line space. In *Computational Line Geometry*, pages 195–222. Springer, 2001.

[32] M Raghavan and B Roth. Solving polynomial systems for the kinematic analysis and synthesis of mechanisms and robot manipulators. *Journal of Vibration and Acoustics*, 117(B):71–79, 1995.

[33] Andrea Russo, Rosario Sinatra, and Fengfeng Xi. Static balancing of parallel robots. *Mechanism and Machine Theory*, 40(2):191–202, 2005.

[34] J Kenneth Salisbury and B Roth. Kinematic and force analysis of articulated mechanical hands. *Journal of Mechanical Design*, 105(1):35–41, 1983.

[35] Yiu Cheung Shiu and Shaheen Ahmad. Calibration of wrist-mounted robotic sensors by solving homogeneous transform equations of the form ax= xb. *Robotics and Automation, IEEE Transactions on*, 5(1):16–29, 1989.

[36] N Simaan and M Shoham. Geometric interpretation of the derivatives of parallel robots jacobian matrix with application to stiffness control. *Journal of Mechanical Design*, 125(1):33–42, 2003.

[37] Nabil Simaan and Moshe Shoham. Singularity analysis of a class of composite serial in-parallel robots. *Robotics and Automation, IEEE Transactions on*, 17(3):301–311, 2001.

[38] Nabil Simaan and Moshe Shoham. Stiffness synthesis of a variable geometry planar robot. In *Advances in Robot Kinematics*, pages 463–472. Springer, 2002.

[39] Nabil Simaan and Moshe Shoham. Stiffness synthesis of a variable geometry six-degrees-of-freedom double planar parallel robot. *The International Journal of Robotics Research*, 22(9):757–775, 2003.

[40] Lung-Wen Tsai. *Robot analysis: the mechanics of serial and parallel manipulators*. John Wiley & Sons, 1999.

[41] Roger Y Tsai and Reimar K Lenz. A new technique for fully autonomous and efficient 3d robotics hand/eye calibration. *Robotics and Automation, IEEE Transactions on*, 5(3):345–358, 1989.

[42] CW Wampler, AP Morgan, and AJ Sommese. Numerical continuation methods for solving polynomial systems arising in kinematics. *Journal of mechanical design*, 112(1):59–68, 1990.

[43] A Wolf, M Shoham, and FC Park. Investigation of singularities and self-motions of the 3-upu robot. In *Advances in Robot Kinematics*, pages 165–174. Springer, 2002.