Kinematic and Experimental Investigation of Manual Resection Tools for Transurethral Bladder Tumor Resection

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Abstract

Background - Transurethral Resection of Bladder Tumors (TURBT) is a challenging procedure partly due to resectoscope limitations. To date, manual resection performance has not been fully characterized. This work characterizes manual resection performance in the bladder while analyzing the effect of resection location on accuracy.

Methods - Kinematic simulations are used to assess kinematic measures of resection dexterity. An experimental protocol for manual resection accuracy assessment is developed. Cross correlations between the theoretical performance measures and the observed experimental accuracy are investigated.

Results - Tangential accuracy correlates relatively strongly with normal singular value and moderately with tangential kinematic conditioning index and tangential minimum singular value. Simulations also clarified difficulties in resecting close to the bladder neck.

Conclusions - Measures to evaluate accuracy and dexterity of TURBT from a kinematic viewpoint are presented to provide a currently missing quantified dexterity baseline in manual TURBT. Limitations in various bladder regions are illustrated.

Keywords: TURBT, Bladder Cancer, Kinematics, Dexterity Evaluation, Resectoscope.

1 Background

Bladder cancer is the fourth most common cancer among men in the US, with an estimated 74,000 new cancer diagnoses, and 16,000 related deaths predicted for 2015 [1]. Transurethral Resection of Bladder

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Tumor (TURBT) is the standard procedure for staging and diagnosis of bladder cancers and treatment of non-muscle invasive tumors (NMIBC). It is typically performed in an outpatient setting under general anesthesia.

During TURBT, a resectoscope is inserted transurethrally to provide access for an endoscope and a cautery loop to reach the surgical site, (Fig. 1). A continuous-flow resectoscope is used to partially distend the bladder by regulating the irrigation inflow and outflow. Figure 2 shows standard resectoscope components. It consists of a telescope lens (30° or 70°), a sheath, a working element and a blind obturator as shown in Fig. 2. The inset in the top right is a close-up view of an electrocautery wire loop. During tumor resection, the cautery loop is inserted distally to the surgeon and beyond the tumor. The cautery loop is then carefully pulled proximally towards the surgeon to scoop out the tumor tissue.

Recent technological developments such as enhanced endoscope designs, new laser techniques, and imaging modules have improved safety and functionality of TURBT. For example, recent imaging modalities include photodynamic diagnosis, narrow-band imaging, optical coherence tomography and confocal laser endomicroscopy [2,3]. Nonetheless, it is still a challenging procedure for surgeons and associated with potential patient morbidity [4,5]. Hindrances to improvement include tool limitations such as lack of intracavitary distal dexterity and in-vivo sensory feedback as well as sparse instrumentation repertoire. Numerous complications have been at least partly attributed to tool limitations such as bladder wall perforations [6], irrigant absorption (due to perforation) [7], bleeding [6] and damage to the ureteric orifices [8].

The quality of the first TURBT procedure greatly influences patient prognosis, treatment follow-ups and costs [3,9]. TURBT guidelines recommend that all visible tumors be removed along with a margin of deeper detrusor muscle for staging [10]. Correct staging is critical as muscle invasive disease requires aggressive management with bladder removal, whereas non-invasive disease can often be managed with endoscopic
resection and intravesical treatments. However, optimal performance has remained elusive. Insufficient resection (under-resection) has been reported in numerous studies in the literature although it has not been investigated closely [3]. In a study by Adiyat and colleagues [9], 70% of patients had visible tumor at the time of restaging TURBT (re-TUR). Among these, 30% were located at the original site. A host of other literature confirm inadequate resection as a challenge in TURBT resulting in understaging, inappropriate treatment regimens, earlier recurrence and likely progression of disease [3,11–15]. Contributing to under-resection is the fact that surgeons have to balance two risks during tumor resection. A shallow resection is likely to leave behind residual tumor. Deep resection can lead to perforation of the bladder, which is a severe complication also potentially spilling tumor into the abdominal cavity.

American Urological Association and European Association of Urology guidelines recommend re-TUR after initial TURBT in any cases where residual or invasive disease is suspected. Repeat TURBT aims to detect residual tumors and to correctly stage tumors by levels of invasiveness. For non-muscle invasive disease, many patients undergo multiple TURBTs because of high recurrence rates. As a consequence, bladder cancer has the highest overall treatment costs per patient among all cancers ranging from 96000 to 187000 US dollars in 2001 [5,14,16].

To overcome some of these challenges, multiple groups including our team have developed robotic systems for transurethral resection [17–20]. We demonstrated a proof-of-concept telesurgical system [20,21] and carried out ex-vivo experiments on bovine bladder to prove its efficacy in targeting different regions of the bladder for both surveillance and intervention [22]. This system is composed of a distal dextrous multi-backbone snake-like robot that is deployed through the urethra and can pass multiple instruments and visualization modules (flexible fiberscope) through its working channels [20,21].

![Figure 2: A Standard 26-Fr Resectoscope: (a) Assembled, (b) Constituent Parts.](image)

TURBT is in general considered successful if the early recurrence rate is low, the tumor is not under-resected allowing accurate staging and no complications occur [23]. These benchmarks are fairly subjective and qualitative. In our earlier study, we proposed a purely kinematic measure to assess TURBT and also to
compare resection accuracy/dexterity in different regions of an assumed spherical bladder model [24]. To the authors’ knowledge, this is the only available investigation in this issue despite the fact that TURBT is a commonly performed urologic surgery. In this paper, we carry out a thorough analytical study of various kinematic measures affecting resection quality. We employ a kinematic computational framework to assess the introduced measures locally (on a bladder point) based on a non-dimensionalized distance as well as regionally (in 16 bladder regions). In order to characterize the measures that are potentially more faithfully descriptive of resection accuracy, we design and perform experiments and then evaluate resection accuracy in normal (depth) and tangential directions. We show that resection accuracy cross-correlates with several kinematic measures. By utilizing the opted kinematic measures based on correlation coefficients, we draw inferences on comparing resection accuracy/dexterity in different regions of the bladder. The results of this study highlight resection dexterity amenability in different bladder regions from a kinematic point of view. More importantly, it provides quantitative measures for resection performance. These measures can be used as benchmarks for novel TURBT system designs such as telerobotic devices.

This article is structured as follows: in section 2.1, the kinematics of resection by a standard resectoscope is modeled as a mechanism with three revolute joints and one prismatic joint (RRRP). The direct and instantaneous kinematics are then derived. Then, a formulation of the Jacobian from hand motion to the resectoscope tip motion subspace is obtained in 2.2. Kinematic measures are introduced and determined analytically. An algorithm is subsequently provided to compute these measures throughout the bladder points as well as regions. In 2.3, the experimental protocol to identify higher correlated kinematic measures is explained and resection variables (measures) are introduced to evaluate accuracy. Sections 3.1 and 3.2 delineate simulation and experiment results in details. The discussion of these results and their implications is addressed in 3.3. We finally concluded this article in section 4 by summarizing the outcomes of the paper and future directions. This paper targets both clinicians and engineers. The involved clinical material does not hinder the latter to gain an understanding of the methods and the contributions. The clinical reader can safely skip the technical parts in sections 2.1 and 2.2.

2 Methods

2.1 Kinematic Modeling of Straight Resectoscope

Modeling Assumptions

The resectoscope is modeled as a four degree-of-freedom (DOF) mechanism. The DOF’s include three rotations (yaw, pitch, roll) and a translational motion along the longitudinal axis of the resectoscope. These configuration variables are expressed as $q_1, q_2, q_3, q_4$ respectively and defined later in this section. Based on measurements from demonstrations by the surgeons coauthoring this paper we have estimated the limits
of first angle to be $|q_1| \leq 38^\circ$ for men and $|q_1| \leq 54^\circ$ for women. Similarly, the second angle is also constrained: $-60^\circ \leq q_2 \leq 50^\circ$.

The bladder is assumed as a sphere of radius $r_b = 50[mm]$ approximately corresponding with a distended bladder with a volume of $600[ml]$ [25]. In reality, a distended bladder is rather blob-like. Nonetheless, the purpose of the measures introduced in this study is to provide a comparative benchmark. Therefore, in this sense, the assumption of a spherical bladder is warranted and facilitates modeling.

In addition, we define the angle $\gamma$ as shown in Fig. 3. This angle depicts the angular location of the point of contact between the cautery loop and the local tangent of the bladder surface ($p$). This angle is measured from the line connecting the center of the cautery loop with the mid-point of the circular cautery segment to the line passing through the center of the cautery loop and the current tissue contact point on the cautery loop (See Fig. 3-a).

During resection, surgeons are inclined to use the loop arc such that $|\gamma| \leq 20^\circ$. This assumption is used in calculation of performance measures while considering the useful resection workspace.

The loop in general has a right angle but some surgeons may prefer to change this angle to facilitate resection in some regions (e.g. the bladder dome). For simplicity, we will consider a right-angle loop throughout. In addition to these assumptions, a fulcrum point located at $\lambda = 30$ mm from the bladder neck (See Fig. 3-b)) is hypothesized based on the mechanics of TURBT practice.

**Frame Assignments and DH Parameters**

Figure 3 illustrates the schematics of a resectoscope. DH parameters are used to identify the kinematics of a resectoscope and six frames are assigned as shown. Figure 3-a also illustrates the tip of the electrocautery loop and the last two frames assigned at the center of the loop arc ($o$) and the perimeter of the arc respectively ($p$).

In Table 1, the pertaining DH parameters are represented. Based on the frame assignment shown in Fig. 3 , $q_1(q_2,q_3)$ is the rotation of $R_1(R_2,R_3)$ about moving frame $\hat{z}_1(\hat{z}_2,\hat{z}_3)$ measured from $\hat{x}_0(\hat{x}_1,\hat{x}_2)$ and $q_4$ is the linear displacement of the resectoscope along the sheath axis $(\hat{z}_4)$ with an offset of $\eta = \lambda + r_b$ from the fulcrum. $\lambda$ is the distance from the fulcrum to the bladder neck (tool entry point), $r_b$ is the bladder radius and $\nu$ is the loop center offset with respect to the loop axis (shown in Fig. 3-a)

**Link Transformations**

The direct kinematics of the resectoscope can be calculated by determining successive homogeneous transformations denoted by $i^{-1}T_i$ from the base frame to the last frame:

$$^0T_5 = ^0T_1^1T_2^3T_2^3T_4^4T_5$$

(1)
Table 1: DH PARAMETERS

<table>
<thead>
<tr>
<th>$i$</th>
<th>$a_{i-1}$</th>
<th>$\alpha_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$q_1$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$\pi/2 + q_2$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$q_3$</td>
</tr>
<tr>
<td>4</td>
<td>$\nu$</td>
<td>0</td>
<td>$\eta + q_4$</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3: Schematics of a Resectoscope: (a) Electrocautery (wire/cutting) loop and the assigned frames (b) Resectoscope model as an RRRP mechanism

This leads to the following transformation:

$$
^0\mathbf{T}_4 = \begin{bmatrix}
    s_1 s_3 & c_3 s_1 & c_1 c_2 & (\eta + q_4) c_1 c_2 \\
    -c_1 s_2 c_3 & +c_1 s_2 s_3 & c_1 c_2 & +\nu s_1 s_3 - c_1 s_2 c_3 \\
    -c_1 s_3 & s_1 s_2 s_3 & c_2 s_1 & (\eta + q_4) s_1 c_2 \\
    -c_3 s_1 s_2 & -c_1 c_3 & c_2 s_3 & -\nu c_1 s_3 + s_1 s_2 c_3 \\
    c_2 c_3 & -c_2 s_3 & s_2 & (\eta + q_4) s_2 + \nu c_2 c_3 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\tag{2}
$$

$$
^4\mathbf{T}_5 = \begin{bmatrix}
    c_\gamma & -s_\gamma & 0 & -r_L c_\gamma \\
    s_\gamma & c_\gamma & 0 & r_L s_\gamma \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\tag{3}
$$

where $r_L$ is the radius of the resection cautery loop and $s_i$, $c_i$ denote $\sin(q_i)$ and $\cos(q_i)$ respectively.

**Instantaneous Kinematics**

Based on observations of transurethral resection procedure, the hand and the imparted loop motions that are involved are mainly translational. As such, in the following sections, transurethral resection is
treated as a task of point contact with the tissue and the focus is on the ability of the surgeon to impart a linear velocity to the resection point \( p \). Thus, in the remainder of the analysis, only the translational components of the Jacobians from configuration space to the tool tip motion space and hand motion space are sought.

The instantaneous kinematics Jacobians for the center of the loop are given by:

\[
J_{o,v} = \begin{bmatrix}

\nu(c_2s_3 + c_3s_1s_2) & -\nu c_1 c_2 c_3 & \nu (c_1 s_2 s_3 + s_1 c_3) & c_1 c_2 \\
-l_c c_2 s_1 & -l_c c_1 s_2 & \nu (s_1 s_2 s_3 - c_1 c_3) & s_1 c_2 \\
+l_c c_1 c_2 & -l_c s_1 s_2 & 0 & s_2 \\
0 & -\nu s_2 c_3 + l_c c_2 & -\nu c_2 s_3 & 0
\end{bmatrix}
\]

(4)

\[
J_{o,\omega} = \begin{bmatrix}

0 & s_1 & c_1 c_2 & 0 \\
0 & -c_1 & s_1 c_2 & 0 \\
1 & 0 & s_2 & 0
\end{bmatrix}
\]

(5)

where \( J_{o,v} \) is the translational Jacobian, \( J_{o,\omega} \) is the rotational Jacobian. Therefore, the total Jacobian

\[
J_o = \begin{bmatrix}

J_{o,v} \\
J_{o,\omega}
\end{bmatrix}
\]

(6)

For the point \( p \) on the loop arc perimeter.

\[
v_p = v_o + \omega_{\text{loop}} \times \rho_{op}
\]

(7)

where \( \omega_{\text{loop}} \) is the angular velocity of the loop, \( \rho_{op} = -r_L \hat{x}_5 \) is the vector from \( o \) to \( p \). \( \hat{x}_5 \) is the unit direction vector of the x-axis of frame \( \{5\} \) described in frame \( \{0\} \). On the other hand:

\[
v_o = J_{o,v} \dot{q}
\]

(8)

\[
\omega_{\text{loop}} = J_{o,\omega} \dot{q}
\]

(9)

\( \dot{q} \in \mathbb{R}^{3\times1} \). Therefore

\[
v_p = J_{p,v} \dot{q}
\]

(10)

where \( J_{p,v} \in \mathbb{R}^{3\times4} \) is the linear velocity Jacobian of the point \( p \) that can be reformulated as

\[
J_{p,v} = S_{p,o} J_o
\]

(11)

where

\[
S_{p,o} = \begin{bmatrix}

I_3 & -(\rho_{op})^\wedge \\
0 & 0
\end{bmatrix}_{3\times6}
\]

(12)
is the linear velocity transformation from \{4\} to \{5\}. \(\cdot\)\(^{\wedge}\) is the cross-product matrix and \(I_3 \in \mathbb{R}^{3 \times 3}\) is an identity matrix. By the same token, the linear velocity Jacobian of the point \(h\) can be determined. Thus,

\[
v_h = J_{h,v} \hat{q}
\]

where \(J_{h,v} \in \mathbb{R}^{3 \times 4}\) is the linear velocity Jacobian of the point \(h\):

\[
J_{h,v} = S_{h,o} J_o
\]

and

\[
S_{h,o} = \begin{bmatrix}
I_3 & -(\rho_{oh})^{\wedge}
\end{bmatrix}_{3 \times 6}
\]

where \(\rho_{oh} = -l \hat{z}_4\) and \(l\) is the resectoscope length is designated at the distance between point \(h\) and point \(o\).

### 2.2 Resection Performance Analysis

In order to quantify manipulation capability of a resectoscope and to compare resection quality in different bladder regions, performance measures should be introduced and computed. An ideal measure (or measures) captures all the factors affecting resection outcome. It is our hypothesis that the major contributing factors are resection dexterity and visual perception. The subject of this study is on the former and fortunately the existing Jacobian-based measures in robotics and mechanisms literature are well-suited for this intent. Herein, we define the subspaces \(S_i = \{v_i \in \mathbb{R}^n \mid \|v_i\| = 1\}\) and \(S_o = \{v_o \in \mathbb{R}^m \mid Jv_i = v_o\}\) as unit-norm input space and output space respectively. Let us iterate the most prominent of such measures in conjunction with these two subspaces. The most common measure is Manipulability criterion [26,27]. It is a measure of the volume of the hyperellipsoid \(v_o\) in output space. Kinematic Conditioning Index (KCI) [28] is another measure that indicates motion isotropy in output space. Both of these measures are directly associated with the singular values of the Jacobian matrix \(J\). Indeed, manipulability is the product of the singular values. KCI is the ratio of the minimum to the maximum singular value and is always between 0 and 1 with larger values showing more isotropy in the output space. Though less common, the minimal singular value is another Jacobian-based measure [29]. It represents the minimal scaling from unit-norm input space to output space.

Our hypothesis is that resection limitations are partly due to dexterity deficiencies of a rigid tool and dexterity is reflected in such measures. Therefore, by exploring these kinematic performance measures, we can identify dexterity limitations and its correlation with resection accuracy. However, the former cannot be directly employed in association with \(J_{h,v}\) or \(J_{p,v}\). This is due to the fact that we have no interest in configuration space to task space (\(\hat{q} \rightarrow v_p\)) or configuration space to hand motion space (\(\hat{q} \rightarrow v_h\)), rather what is of interest is relating instantaneous linear motions in hand motion space (analogous to unit-norm
input space) to the linear motions in the task space (analogous to output space) \((v_h \rightarrow v_p)\). In this regard, the velocity transformation from the former to the latter should be determined. Based on Eqs. (13) (10),

\[
v_p = J_{p,v} J_{h,v}^T v_h
\]  

(16)

where \(J_{h,v}^T\) is Moore-Penrose pseudo-inverse of \(J_{h,v}\) and equals [30]:

\[
J_{h,v}^T = J_{h,v}^T (J_{h,v} J_{h,v}^T)^{-1}
\]

(17)

For brevity, let us define

\[
J = J_{p,v} J_{h,v}^T
\]

(18)

Having determined \(J\), multiple kinematic measures are assessed in tangential and normal directions. Which of such measures are more relevant are established by properly designed experiments explained in section 2.3. In the following subsections, the kinematic measures are derived analytically and discussed. Also an algorithm is proposed that is used to compute these measures.

**Kinematic Dexterity Measures**

In tangential dexterity/accuracy evaluation, we are interested in hand motions causing task-space motions tangential to the bladder surface. Therefore a transformation is sought that maps a particular subspace of hand motions -the one causing tangential motions in task space, to the subspace of task-space tangential motions. In normal dexterity evaluation, a hand-space-to-task-space mapping is sought that causes task space motions normal to the sphere. In this case, hand and task subspaces of interest are both lines. To determine the normal task space motion mapping, the normal vector to the sphere surface should be calculated at each point. Using spherical coordinates \((\rho, \theta, \phi)\) on frame \(\{b\}\) to locate each sphere point, the normal vector is

\[
\hat{e}_{\rho} = [\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi)]^T
\]

(19)

as shown in Fig. 4. Frame \(\{b\}\) is a translation of \(\{0\}\) to the sphere center. The projection matrix that maps \(v_p\) on the normal direction is

\[
P_n = \hat{e}_{\rho} \hat{e}_{\rho}^T
\]

(20)

where \(P_n \in \mathbb{R}^{3 \times 3}\). Substituting for \(\hat{e}_{\rho}\) from (19) in (20)

\[
P_n(\theta, \phi) = \begin{bmatrix}
c_\theta^2 s_\phi^2 & c_\theta c_\phi s_\phi & c_\theta s_\phi c_\phi \\
 s_\theta c_\phi s_\phi & s_\theta^2 s_\phi^2 & s_\theta s_\phi c_\phi \\
 c_\theta s_\phi c_\phi & s_\theta s_\phi s_\phi & c_\phi^2
\end{bmatrix}
\]

(21)

Therefore, pre-multiplying both sides of (16) by \(P_n\)

\[
v_{p,n} = J_n v_h
\]

(22)
where \( v_{p,n} = P_n v_p, J_n = P_n J \). Note that \( \text{rank}(J_n) = 1 \). The normal singular value is defined herein as

\[
\sigma_n \triangleq \sigma(J_n)
\]  

(23)

For tangential direction, pre-multiplying both sides of (16) by \( P_t \) would yield

\[
v_{p,t} = J_t v_h
\]  

(24)

where \( v_{p,t} = P_t v_p, J_t = P_t J \) and \( P_t \) is the complementary projector to \( P_n \) and thus \( P_t = I - P_n \) \cite{31}.

Determining \( J_t \), the following tangential measures are defined:

\[
\sigma_{\text{min},t} \triangleq \sigma_{\text{min}}(J_t), KCI_t \triangleq \frac{\sigma_{\text{min},t}}{\sigma_{\text{max},t}}, \mu_t \triangleq \sigma_{\text{min},t} \sigma_{\text{max},t}
\]  

(25)

where \( \sigma_{\text{max},t} \triangleq \sigma_{\text{max}}(J_t) \)

Such measures are pointwise in that the evaluation is performed on one single point in task space (though for a range of practical points on the cautery loop). To determine performance in an area, these measures would be integrated on that area and divided by the area to yield an average value. Accordingly,

\[
\bar{\sigma}_{\text{min},t} = \frac{\int_A \sigma_{\text{min},t} ds}{\int_A ds}, KCI_t = \frac{\int_A KCI_t ds}{\int_A ds}, \bar{\mu}_t = \frac{\int_A \mu_t ds}{\int_A ds}
\]  

(26)

where \( A \) represents the area of interest.

Algorithm for Kinematic Dexterity Measures Evaluation

Algorithm 1 presents the calculation of the kinematic measures in the bladder spherical model. This procedure starts with finding \( q^{(j)}_t \), the \( j \)th inverse kinematic solution (See Appendix A). Unit tangential/normal velocity is assumed at the resection point \( p \). The instantaneous kinematic equation (16) is then used to obtain \( v_h \). After this step, the Euclidean norm of \( v_h \) is calculated and the original \( v_p \) is scaled by the...
reciprocal of the norm. These scaled vectors would form the manipulability ellipse on the given plane. The ratio of the minimum to the maximum norms would yield the local $KCI_t$ and their product yields local $\mu_t$. To determine $\sigma_n$, the unit velocity in the direction normal to the sphere surface is calculated in world coordinate frame $\{0\}$ for each point. Then hand velocity is computed from equation (16). The inverse of the Euclidean norm of this velocity is computed and denoted by $\sigma_n$. Finally, the average of the measure under consideration among all resection points on the loop ($-20 \leq \gamma \leq 20$) and inverse kinematic solutions ($q_i^{(j)}$) is determined as the kinematic measure at point $p$.

This algorithm determines pointwise measures. To quantify performance within the bladder half-octant, eq. (26) is applied. The discretization of these equations over an area results in an average over discrete points in that area. Thus, to compare these measures in different regions of a bladder, the sphere model is divided into 16 equal zones, each 45° apart in both azimuth and altitude as demonstrated in Fig. 5. These zones are numbered 1-16 for identification and called half-octants (H/O) throughout the rest of this article. In this figure, the bladder dome is situated at the superior area and the anterior/posterior area is to the positive/negative $\hat{z}_b$ direction.

![Figure 5: Bladder half-octants: (a) Posterior hemisphere, (b) Anatomical directions (c) Anterior hemisphere](image)

Next, all measures are computed at uniformly distributed points at all 16 half-octants. Subsequently, the average values are determined and reported per half-octant using Algorithm 1. These values are shown in Table 3.

### 2.3 Experimental Investigation

There are multiple reasons that motivate an experimental investigation into manual resection by the current standard resectoscopes used in TURBT. First, it is desired to explore correlations between the proposed
Algorithm 1: Computation of Kinematic Measures

**Input:** Given point \( \mathbf{p} \) on sphere

for \(-20^\circ \leq \gamma \leq 20^\circ\) do

for \( q_i^{(j)} \) inverse kinematic solution do

Specify \( \left[ \mathbf{V}_{p,t} \right]_3 \times n = [v_{p,t}^{(1)}, v_{p,t}^{(2)}, \ldots, v_{p,t}^{(n)}] \) where \( v_{p,t}^{(i)} = [\cos(\alpha_i), \sin(\alpha_i), 0]^T \) and \( 0 \leq \alpha_i \leq 2\pi \)

Specify \( \mathbf{v}_{p,n} = [1, 0, 0]^T \)

Calculate \( \left[ \mathbf{V}_{p,t} \right]_3 \times n = [\mathbf{v}_{p,t}^{(1)}, \mathbf{v}_{p,t}^{(2)}, \ldots, \mathbf{v}_{p,t}^{(n)}] \) located on the desired plane and \( \mathbf{v}_{p,n} \), unit velocity at point \( \mathbf{p} \) normal to the sphere surface where \( \mathbf{v}_{p,t}^{(i)} = \frac{1}{4} \mathbf{R_v}_{p,t}, \mathbf{v}_{p,n} = \frac{1}{4} \mathbf{R_v}_{p,n} \) and \( \mathbf{R} \) is the rotation transformation from \( \{0\} \) to \( \{4\} \)

Calculate \( \mathbf{V}_{h,t} = \mathbf{J}_{h,v} \mathbf{J}_{p,v}^T \mathbf{V}_{p,t}, \mathbf{v}_{h,n} = \mathbf{J}_{h,v} \mathbf{J}_{p,v}^T \mathbf{v}_{p,n} \) where \( \left[ \mathbf{V}_{h,t} \right]_3 \times n = [\mathbf{v}_{h,t}^{(1)}, \mathbf{v}_{h,t}^{(2)}, \ldots, \mathbf{v}_{h,t}^{(n)}] \)

Calculate \( m^{(i)} \approx \|v_{h,t}^{(i)}\|_2 \)

Determine \( \sigma_{\text{min},t} = \max(m^{(i)}), \sigma_{\text{max},t} = \min(m^{(i)}) \)

Determine \( KCI_t = \frac{\sigma_{\text{min},t}}{\sigma_{\text{max},t}}, \mu_t = \sigma_{\text{min},t} / \sigma_{\text{max},t} \)

Determine \( \sigma_n = \frac{1}{\|v_{h,n}\|_2} \)

end for

end for

**Output:** average \( \sigma_{\text{min},t}, KCI_t, \mu_t, \sigma_n \)

kinematic measures and resection quality to find which kinematic measures correlate with resection performance. Second, we wish to verify chronic under-resection in TURBT as it has not been well studied though the incidence is well documented (e.g. [3], [9]). In addition, a comparison of real resection accuracy in different bladder areas gives insight into comparative resectability of different bladder regions. In addition, we attempt to provide a standard experimental protocol that can be used by designers to compare the efficacy of any new device (robots for transurethral resection e.g. [20, 21, 32]) against a standard resectoscope or any other tool. Hence, An experimental protocol for TURBT in multiple points of a hypothetical sphere in space was designed and implemented. The following subsections present the experimental protocol.

**Experimental Procedure**

TURBT phantoms were created using a standard Ø60[mm] × 15[mm] petri dish filled with a mixture of 23[gr/L] agar (Sigma-Aldrich #A7002-250G) and 110[gr/L] milk in distilled water. Milk was used to eliminate any potential visual cue of the sample depth by providing opacity. Then, a disk of 13.65[mm] in diameter and 6[mm] in height was placed in the center of the agar mixture on top of its surface. The disk was subsequently removed after the mixture was cured to leave a cavity for the mock-up lesion. The lesion was made out of the same agar mixed with red glitter which was poured into the cavity and allowed to cure at room temperature. The final product was a matte agar gel with a colored lesion in the center top as shown in Fig.6-a.
Figure 6: sample (a) before resection, (b) after resection, (c) resected volume

Considering the geometric symmetry of the bladder sphere relative to the tool kinematics, it can be inferred that all kinematic measures are solely functions of the distance from the bladder neck. A non-dimensionalized distance $\mathbf{x} = \frac{x - \lambda}{r_b}$ is adopted here where $x$ is the x-component of $\mathbf{p}$ in frame {0} (See Fig. 3). Several points on a hypothetical sphere at various distances are selected including $\mathbf{x} = 0.4, 0.6, 0.8, 1.0, 1.3, 1.7, 2.0$. It was not possible to perform resection on regions with $\mathbf{x} < 0.4$ due to experimental set-up space constraints. To place the petri dishes at these locations, a PUMA 560® robot was used. Automatic C-code generation was done by MATLAB Real-Time Simulink® and real-time implementation was carried out by MATLAB xPC Target™. A suitable location in the workspace of the robot was identified as the bladder center based on typical patient bladder location with respect to the surgeon in an operating room. The desired locations represented as end-effector poses (end-effector center position and orientation) were obtained in world coordinate frame through successive rotation matrices. The corresponding robot joint variables were computed by applying the inverse kinematics solution of the PUMA 560 according to [33]. Then the robot was commanded to servo to the desired pose by means of a gravity-compensated PD (proportional-derivative) control in joint space.

Fig. 7 represents the experimental setup. It was designed to mimic the clinical TURBT procedure as closely as feasible. Therefore, the overall layout of the setup and the associated geometric dimensions were selected with the clinical settings in mind. As such, the constraint point for the resectoscope was assumed $30[mm]$ off from the bladder neck ($\lambda = 30$). A flexible ring on an adaptor was mounted on a tripod with adjustable height and tilting head. The ring provided a soft pivot point for the resectoscope similar to the real anatomy. A cardboard panel was mounted on the adaptor and a drape was laid over the robot end-effector during resections to prevent any visual cue of the sample depth. Therefore, the surgeon could only observe the surface of the phantoms by a standard endoscopic lens which displayed the resection site on a standard endoscopy monitor display.
Three surgeons were asked to resect the entire lesion along with 3\(\text{mm}\) margin in all directions (tangential and normal) using a 26Fr Storz resectoscope. In other words, the surgeons were required to remove a cylinder of slightly larger dimensions than the red lesion. This was to help simulate the clinical circumstances where the tumor boundaries were not generally visible and the surgeons were recommended to remove tumors with marginal tissue in order to help tumor staging. The order of presentation of samples was randomized to avoid biasing the results by multiple successive trials. The surgeons were not provided with any information as to the quality of their resections to eliminate learning and thus potentially biasing the results.

After completion of resections (Fig. 6-b), the cavity was first filled and allowed to cure. The cured cast was then removed from the surrounding agar. There was no diffusion of the two agar into one another as observed by the different colors of the original agar (pale white due to use of milk when preparing the agar) and the later translucent agar. Powder talc was then applied to the surface of the removed agar with a brush (Fig. 6-c) and then it was scanned using a 3D laser scanner (FaroArm Fusion). Talc helps block the body reflection of the translucent agar that can cause undulated inaccurate surface scans. The repeatability of the scanning device is a maximum 0.104\(\text{mm}\).

Geomagic\textsuperscript{®} was used for the 3D render and record of the scan. Geomagic can display the point cloud scanned by FaroArm and can also produce surfaces and volumes. In turn, the scanned points can be exported to various standard CAD formats.

Resection Accuracy Measures

Table 2 enumerates and defines the resection measures that are henceforth used to investigate the
accuracy of each resection trial. In this table, \( p, q \) denote the number of points selected for tangential and normal mean computations, respectively. \( z_{ref} \) and \( r_{ref} \) is the resection desired depth and radius based on the lesion depth and radius along with 3mm margin.

**Resection Measures Computation**

The scan data was exported as *stl* files to be processed by a code written in MATLAB® that computed the resection measures in Table 2.

The method used to calculate resection measures is as follows. First, The bottom boundary of the removed agar (sample top surface boundary) was sought at a height level of \( 0.1 – 0.2[mm] \) max from the bottom. This is necessary since the bottom edge of the sample is not distinguishable as the sharp edges do not reflect the laser beam back at the same direction and are therefore 'lost' in the scan. The selected bottom surface was then meshed by a rectangular grid. The mesh resolution size was chosen such that it produced approximately 2000 grid cells. For resection normal measures, the average depth of the points inside each grid cell was determined and used as a representative of the respective cell \( i \). These values were stored and used to compute the normal resection measures. For tangential measures, after selection of the bottom surface, the model was sectioned to twenty horizontal slices. Each slice was divided into 10-degree sectors. Next, the average of the points were calculated per sector. These average points were stored and used to determine the tangential resection measures.

### Table 2: Resection Measures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_t )</td>
<td><strong>Tangential Mean:</strong> average of the radial distances over all resection points of the resected volume</td>
<td>( (1/p) \sum_{i=1}^{p} r_i )</td>
</tr>
<tr>
<td>( E_t )</td>
<td><strong>Tangential Error:</strong> average radial errors over all resection points of the resected volume, ( r_{ref} = 9.82[mm] )</td>
<td>( r_{ref} - M_t )</td>
</tr>
<tr>
<td>( M_n )</td>
<td><strong>Normal Mean:</strong> average of resection points depths</td>
<td>( (1/q) \sum_{i=1}^{q} z_i )</td>
</tr>
<tr>
<td>( E_n )</td>
<td><strong>Normal Error:</strong> average of resection points depth errors, ( z_{ref} = 9[mm] )</td>
<td>( z_{ref} - M_n )</td>
</tr>
</tbody>
</table>
Figure 8: Projections of the tangential manipulability ellipses on the right lateral bladder hemisphere. Bottom right inset provides anatomical context

3 Results

3.1 Simulation Results

Algorithm 1 was implemented by a code in MATLAB\textsuperscript{\textregistered}. The simulation parameters were selected similar to a real cautery loop deployed in a straight resectoscope as follows: $l = 500$[mm], $\nu = 3.75$[mm], $r_L = 5.2$[mm]. Other simulation parameters were presented in section 2.1.

Fig. 8-b presents the tangential manipulability ellipses on the right lateral bladder hemisphere. These ellipses were calculated based on Algorithm 1. It is noted that the surgeon inserts the resectoscope through the bladder neck (base inferior) toward the positive $\hat{x}_b$ direction. Fig. 8-a and 8-c are the side-view projections of the left and right quadrants of the same hemisphere. Note the higher eccentricity of ellipses in the inferior right quadrant (Fig. 8-a) in comparison to superior right quadrant (Fig. 8-c). Higher eccentricity corresponds to reduced isotropy of kinematic dexterity and diminished $KCI_t$.

The average kinematic measures in each H/O are reported in Table 3. This table helps compare the dexterity of manual resection in various regions.

Fig. 9 presents plots of the four introduced kinematic measures with respect to the non-dimensionalized distance $\Xi$. Note that the tool workspace is also drawn using assumptions in section 2.1, however it is not
Table 3: MEAN TRANSLATIONAL KCI, MANIPULABILITY, MIN. SINGULAR VALUE AND NORMAL SINGULAR VALUE IN BLADDER HALF-OCTANTS

<table>
<thead>
<tr>
<th>H/O num.</th>
<th>KCI/μt</th>
<th>σmin,t/σn</th>
<th>H/O num.</th>
<th>KCI/μt</th>
<th>σmin,t/σn</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.83/0.11</td>
<td>0.30/0.54</td>
<td>9</td>
<td>0.83/0.11</td>
<td>0.30/0.54</td>
</tr>
<tr>
<td>2</td>
<td>0.68/0.10</td>
<td>0.26/0.35</td>
<td>10</td>
<td>0.68/0.10</td>
<td>0.26/0.35</td>
</tr>
<tr>
<td>3</td>
<td>0.41/0.10</td>
<td>0.20/0.22</td>
<td>11</td>
<td>0.41/0.10</td>
<td>0.20/0.22</td>
</tr>
<tr>
<td>4</td>
<td>0.38/0.08</td>
<td>0.14/0.17</td>
<td>12</td>
<td>0.38/0.08</td>
<td>0.14/0.17</td>
</tr>
<tr>
<td>5</td>
<td>0.38/0.08</td>
<td>0.14/0.17</td>
<td>13</td>
<td>0.38/0.08</td>
<td>0.14/0.17</td>
</tr>
<tr>
<td>6</td>
<td>0.41/0.10</td>
<td>0.20/0.22</td>
<td>14</td>
<td>0.41/0.10</td>
<td>0.20/0.22</td>
</tr>
<tr>
<td>7</td>
<td>0.68/0.10</td>
<td>0.26/0.35</td>
<td>15</td>
<td>0.68/0.10</td>
<td>0.26/0.35</td>
</tr>
<tr>
<td>8</td>
<td>0.83/0.11</td>
<td>0.30/0.54</td>
<td>16</td>
<td>0.83/0.11</td>
<td>0.30/0.54</td>
</tr>
</tbody>
</table>

Table 4: CORRELATION COEFFICIENTS AND %95 CONFIDENCE INTERVALS FOR RESECTION MEASURES AGAINST KINEMATIC MEASURES

<table>
<thead>
<tr>
<th></th>
<th>σmin,t</th>
<th>KCI/μt</th>
<th>μt</th>
<th>σn</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_t</td>
<td>-0.56(-0.72,-0.31)</td>
<td>-0.52(-0.69,-0.25)</td>
<td>-0.09(-0.41,0.22)</td>
<td>-0.64(-0.79,-0.44)</td>
</tr>
<tr>
<td>E_n</td>
<td>-0.32(-0.59,0.09)</td>
<td>-0.34(-0.60,0.05)</td>
<td>-0.17(-0.08,0.40)</td>
<td>-0.23(-0.52,0.26)</td>
</tr>
</tbody>
</table>

presented here for brevity. It is verified that the assumed distended bladder model always lies within the tool workspace. Hence, configuration limits do not affect simulation results.

3.2 Experimental Results

Figure 10-a shows box plots of normal and tangential mean versus non-dimensionalized distance in 40 resection trials by the surgeons. The red line, the + symbol and the small circle represent the median, the mean and outlier data. The red and the blue dashed line marks the desired resection depth and radius respectively (z_{ref} = 9[mm], r_{ref} = 9.82[mm]). Fig. 10-b shows the errors in the same trials.

Table 4 shows correlation coefficients and %95 confidence intervals for resection measures against tangential and normal measures in the experimental trials. The confidence intervals brought in parentheses are obtained by 5000 bias-corrected and accelerated (BCa) bootstrap sample sets [34]. Stronger results are boldfaced.
Figure 9: Plots of (a) $KCI_t$(black) and $\mu_t$(blue), (b) $\sigma_{\text{min},t}$(black) and $\sigma_n$(blue) vs non-dimensionalized distance $\bar{x}$.
Figure 10: Plots of (a) $M_n$ (normal mean) and $M_t$ (tangential mean), (b) $E_n$ (normal error) and $E_t$ (tangential error) vs non-dimensionalized distance $\bar{x}$. 
3.3 Discussions

Examining the box plots in Figs. 10 confirms that the surgeons almost always under-resect in both tangential and normal directions. More specifically, normal and tangential error medians for the selected distances are \{3.85, 4.28, 3.65, 2.96, 2.42, 3.20, 2.58\} and \{1.82, 1.09, 1.62, 1.39, 1.88, 0.68, 0.48\} respectively. This agrees with the chronic under-resection as hypothesized in previous literature (e.g. [3], [9]).

In regards to tangential accuracy, the results in Table 4 suggest that it has a relatively strong correlation with \(\sigma_n\) ((-0.79,-0.44)) and \(\sigma_{\text{min},t}\) ((-0.72,-0.31)) and medium correlation with \(KCI_t\) ((-0.69,-0.25)). Hence, all these measure could be suited kinematic measures. The correlation between tangential error and normal singular value may seem counter-intuitive. However, it is important to note that tangential error computation is performed throughout the depth. From a different perspective, all singular values of manipulability ellipsoid i.e. \(\sigma_{\text{min},t}, \sigma_{\text{max},t}, \sigma_n\) may be candidates for manipulability - tangential or overall and therefore affect resection accuracy - in normal or tangential directions. Amongst these, \(\sigma_n\) has the largest variation with respect to distance in simulations (0-1, See Fig. 9-b). This can explain why it may represent higher correlation with respect to distance in simulations (0-1, See Fig. 9-b). This can explain why it may

The correlation coefficient between normal error and kinematic measures are also reported in Table 4 although the confidence intervals indicate medium, little or no linear correlation. This is justified by insufficient depth perception. In other words, the surgeon cannot determine properly how deep he/she is resecting. This alludes to the importance of visualization enhancement in TURBT. On the other hand, Table 4 shows there is a weak correlation between tangential manipulability and resection accuracy. This implies tangential manipulability cannot capture resection accuracy and therefore is not a proper measure. In fact, simulation results in Fig. 9-a shows that tangential manipulability varies between 0 – 0.14 and for the most part of the distance span \(0.3 \leq \pi \leq 2\), it’s 0.08 – 0.14. This small variation is responsible for low correlations. This can also be verified by inspecting H/O values in Table 3 where the four H/O’s 1-4 (also 8-5,9-12,16-13) have only slightly different tangential manipulability values of 0.11, 0.10, 0.10, 0.08.

Based on Table 3, the mean \(KCI_t\) in H/O’s 1-4 is 0.83, 0.68, 0.41 and 0.38 respectively. This is in a decreasing order from the bladder dome to the bladder neck with a highest to lowest ratio of about two. The same values hold for half-octants 8-5,9-12,16-13. The values for this series of H/O’s are equal due to the sphere symmetry. Overall, it is inferred that average \(KCI_t\) over the lateral walls almost doubles as H/O’s go further away from the bladder neck (tool entry point) and closer to the bladder dome zone. The normal and minimum tangential singular values exhibit a decreasing trend as well (0.54 – 0.17 and 0.30 – 0.14 respectively). Indeed, \(\sigma_n\) and \(\sigma_{\text{min},t}\) are approximately 335% and 214% higher in the dome zone than in the neck zone. Though the rate of change is lower for \(\sigma_{\text{min},t}\). It is noteworthy that tumors are generally said to be difficult to resect at bladder dome, anterior and neck. Nevertheless, this is not
necessarily attributed to kinematic limitations. In fact these areas may be too far to reach when the bladder is distended (anterior/dome) or out of the field of view (anterior/dome/neck) [3]. Hence, improved visualization technologies or depth perception enhancement should be beneficial in these zones.

A close examination of Fig. 9-a suggest that $KCI_t$ decreases at a steep slope until it reaches a minimum of 0.16 at $\overline{x} = 0.36$. Moving further away, it increases at a lesser slope until it matches unit $KCI_t$ at bladder dome ($\overline{x} = 2$). This trend is also identifiable by analyzing the eccentricity of the ellipses plotted in Fig. 8. In detail, let us take a series of ellipses in red. From left where $\overline{x} = 0$, the ellipse is a small circle implying unit $KCI_t$ with fairly small singular values. Toward positive $\overline{x}$, the eccentricity begins to increase corresponding to lower $KCI_t$ until a specific point ($\overline{x} = 0.36$). After that, the ellipses gradually become more isotropic until $\overline{x} = 2$ where full isotropy is achieved.

The minimum/maximum/average $KCI_t$ is 0.56/1/0.80 at the Superior and 0.16/1/0.37 at the inferior hemisphere. Therefore, the superior hemisphere has higher kinematic feasibility for resection (almost 218% higher $KCI_t$ on average). This can be also concluded by comparing eccentricity of the ellipses in Fig. 8-a and Fig. 8-c. The average $KCI_t$ over the entire bladder is 0.59.

Normal singular value falls sharply from 1.00 to 0.11 at $\overline{x} = 0.09$, but increases steadily until it reaches unity at $\overline{x} = 2$. The reason for the sharp fall is the location of the fulcrum point $\mathbf{f}$ to the left of the bladder neck and the obtuse angle between the resectoscope axis and the bladder surface normal directions at areas close the bladder neck. On the other hand, $\sigma_{min,t}$ represents a steady increase from 0.07 to 0.35 versus the non-dimensionalized distance $\overline{x}$.

4 Conclusion

TURBT is a minimally-invasive surgical procedure used to diagnose bladder cancer and treat non-muscle invasive bladder cancer where a surgeon removes visible tumor with an electrosurgical loop that cuts the tumor out. This paper mainly addresses the kinematic measures in this procedure. After modeling the kinematics, several Jacobian-based measures are proposed including tangential kinematic conditioning index, tangential manipulability, tangential minimum singular value and normal singular value. Experimental trials simulating clinical TURBT is performed by three surgeons and the resection accuracy in tangential and normal(depth) directions are measured. It is verified that the surgeons generally under-resect. On further investigation, it is shown that tangential accuracy correlates relatively strongly with normal singular value and moderately with tangential kinematic conditioning index and tangential minimum singular value. A weak linear correlation with tangential manipulability is substantiated hence disqualifying it as a measure. On the other hand, normal resection accuracy is demonstrated to be weakly correlated with any of the kinematic measures suggesting that other factors may influence this variable. It it our judgment
that visualization enhancement should considerably improve depth resection outcomes.

The certified measures are utilized to compare kinematic accuracy/dexterity locally in all bladder points based on the distance from the bladder neck and regionally in 16 bladder zones.

This study has few limitations. The cylindrical lesion assumption with 3[mm] margins for resection procedure is a simplification of clinical TURBT procedure. In reality, bladder tumors can have many different appearances. Often times, they appear as tissue lumps that protrude from bladder wall. However, the adopted model in this study is the most viable model that could be actualized in a repeatable way. Also, this model does not alter the kinematics of resection procedure. In addition, the resection study was limited in scope and size by focusing on a small number of experienced urologists. This limitation stems from a realization that the number of resections and effort involved in analyzing each resection sample limit the ability to expand this study to span a large number of study subjects. We therefore focused on only experienced surgeons in order to eliminate potential confounding factors. Nevertheless, we believe this study reports a first attempt at correlating theoretical measures for resection dexterity with experimental data. As such, it contributes a first effort that we hope will seed and power future studies in this area.

At a prospective endeavor, the introduced measures will be evaluated on a multi-backbone snake-like continuum robot for bladder resection. It is anticipated that the continuum robot will yield higher kinematic measures hence rendering more accurate resections.

A Inverse Kinematic Solutions

To solve the inverse kinematic problem for a point on the loop, given $\gamma$ and $^0T_5$, $^0T_4$ is sought. Using successive transformations Eq. (1),

$$^0T_4 = ^0T_5 \, ^5T_4^{-1}$$

where $^5T_5$ is given by Eq. (3). Therefore,

$$^5T_5^{-1} = \begin{bmatrix}
    c_\gamma & s_\gamma & 0 & r_L \\
    -s_\gamma & c_\gamma & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}$$

Now, algebraic method is utilized to determine solutions given $^0T_4$

$$^0T_4 = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} & p_x \\
    r_{21} & r_{22} & r_{23} & p_y \\
    r_{31} & r_{32} & r_{33} & p_z \\
    0 & 0 & 0 & 1
\end{bmatrix}$$
where all matrix elements are brought in Eq. (2). Since \( r_{13} = c_1c_2 \) and \( r_{23} = s_1c_2 \), therefore \( c_1 = \frac{r_{13}}{\pm \sqrt{1 - r_{33}^2}} \) and \( s_1 = \frac{r_{23}}{\pm \sqrt{1 - r_{33}^2}} \). This gives \( q_1 \)

\[
q_1 = \text{atan2}(\frac{r_{23}}{\pm \sqrt{1 - r_{33}^2}}, \frac{r_{13}}{\pm \sqrt{1 - r_{33}^2}}), \text{ if } r_{33} \neq \pm 1
\]

(30)

from \( r_{33} = s_2 \),

\[
q_2 = \text{atan2}(r_{33}, \pm \sqrt{1 - r_{22}^2})
\]

(31)

Since \( r_{32} = -c_2s_3 \) and \( r_{31} = c_2c_3 \), therefore \( s_3 = \frac{r_{23}}{c_2} = \frac{-r_{23}}{\pm \sqrt{1 - r_{33}^2}} \) and \( c_3 = \frac{r_{31}}{c_2} = \frac{r_{31}}{\pm \sqrt{1 - r_{33}^2}} \). This yields \( q_3 \)

\[
q_3 = \text{atan2}(\frac{-r_{32}}{\pm \sqrt{1 - r_{22}^2}}, \frac{r_{31}}{\pm \sqrt{1 - r_{33}^2}}), \text{ if } r_{33} \neq \pm 1
\]

(32)

In case \( r_{33} = \pm 1 \), there are infinite solutions for \( q_1 \) and \( q_3 \). The solution are \( q_1 + q_3 = \text{atan2}(r_{12}, -r_{22}) \) or \( q_1 - q_3 = \text{atan2}(r_{12}, -r_{22}) \). As for \( q_4 \),

\[
\begin{align*}
q_4 &= \frac{p_y - \nu r_{11}}{r_{13}} - \eta, \text{ if } r_{13} \neq 0 \\
q_4 &= \frac{p_y - \nu r_{21}}{r_{23}} - \eta, \text{ if } r_{23} \neq 0 \\
q_4 &= \frac{p_y - \nu r_{31}}{r_{33}} - \eta, \text{ if } r_{33} \neq 0
\end{align*}
\]

(33)

Acknowledgement

This research was supported by NIH grant R21EB015623-01A1.

Conflict of Interest

Nabil Simaan is co-founder of a new start-up seeking to develop new technologies for robot-assisted TURBT. In addition he holds several patents in the area of surgical robotics. None of these patents or industry relations have been allowed to bear any impact on this research. Dr. Simaan’s activity was reviewed by Vanderbilt University conflict of interest committee and a conflict of interest management plan was set in place in accordance with PHS policy.

References


